

Corrigendum to “Income distribution and housing prices: An assignment model approach”

[J. Econ. Theory 151 (2014) 381–410]

Niku Määttänen ^{a,b}, Marko Tervio ^{c,b,*}

^a *University of Helsinki, Finland*

^b *Helsinki GSE, Finland*

^c *Aalto University, School of Business, Finland*

October 2026

Abstract

The proof of Lemma 1 (positive assortative matching) in Appendix A of the paper relies on the claim that a certain condition holds for every quasi-concave utility function. The claim is incorrect. The condition is that housing be a normal good, and without normality Lemma 1 can fail. We replace the assumption of strict quasi-concavity by the assumption that housing is a normal good. All results of the paper hold under the amended assumption, and the empirical application, which uses CES utility, is unaffected. This note provides a revised Appendix A, which replaces the original one.

1. The error

Lemma 1 states that in equilibrium there is positive assortative matching (PAM) by household wealth and house quality. Its proof in Appendix A reduces the claim to condition (25), $u_{xy} - u_{yy}u_x/u_y \geq 0$, and then asserts that this condition holds for any quasi-concave utility function. That assertion is incorrect. Condition (25) says that the marginal rate of substitution $MRS_{xy} = u_x/u_y$ is nondecreasing in y , i.e., that housing is a normal good. Quasi-concavity is a different property: it says that MRS_{xy} decreases along an indifference curve. Neither property implies the other.

The intended assumption is that housing is a normal good. The paper states this in the Introduction (p. 382) and in Section 2 (p. 384), and the proof of Proposition 4 (p. 391) uses it (u_x/u_y is increasing in θ). It was, however, missing from the assumptions in Section 3, where strict quasi-concavity stood in its place. Quasi-concavity plays no role in the model: houses are indivisible, so budget sets need not be convex, and the arguments rely only on the single-crossing property implied by normality (Lemma A.1 below). We therefore replace strict quasi-concavity by normality (Section 2 below). Since neither property implies the other, the amended assumptions are neither weaker nor stronger than the original ones. The CES utility function (7) used in the empirical application satisfies both for every $\rho < 1$; in particular, it satisfies normality because $u_x/u_y = \frac{\alpha}{1-\alpha}(y/x)^{1-\rho}$. The proof of existence of equilibrium in Appendix A also relied on normality without assuming it.

*Corresponding author.

E-mail addresses: niku.maattanen@helsinki.fi (N. Määttänen), marko.tervio@aalto.fi (M. Tervio).

The revised Appendix A below replaces the original in its entirety. It gives a short proof of Lemma 1 that uses only the households' budget sets, so it also covers the two-sided model of Section 3.4, and a complete proof of existence. It also characterizes the equilibria of the economy with discretized house types, which establishes the claims about equilibrium prices made in Section 3 and in the original appendix.

2. Changes to the main text

1. Section 3, p. 386, first paragraph: replace “We assume standard preferences: u is strictly increasing, differentiable, and strictly quasi-concave.” by

“We assume standard preferences: u is strictly increasing and differentiable, and housing is a strictly normal good.”

2. Section 3, p. 387, paragraph following Lemma 1: replace “In short, the diminishing marginal rate of substitution guarantees PAM” by “In short, the normality of housing guarantees PAM”.
3. Section 5.3, p. 398, paragraph following (20): replace “(as long as it exhibits the diminishing MRS required for positive sorting)” by “(as long as housing is a normal good, as required for positive sorting)”.
4. Section 6, p. 403, first paragraph: replace “due to concave utility,” by “because housing is a normal good,”.
5. Appendix A, pp. 405–407: replace by the revised Appendix A below.

The property used in the proof of Proposition 4 is now part of the assumptions, and no other changes are needed.

Appendix A. Proofs

Proof of Lemma 1: Positive assortative matching by wealth and house quality Throughout this appendix, u satisfies the assumptions of Section 3, which we use in the following form: u is continuous; wherever $y > 0$, it is strictly increasing and differentiable, with $u_y > 0$; and housing is a strictly normal good, in the sense that the marginal rate of substitution between house quality and other goods, $MRS_{xy}(x, y) = u_x(x, y)/u_y(x, y)$, is strictly increasing in y . For house types $x < x'$ and consumption $y \geq 0$, define the reservation price $\Delta(x'|x, y)$ by

$$\Delta(x'|x, y) = y - \min\{t \geq 0 : u(x', t) \geq u(x, y)\}, \quad (\text{A.1})$$

so that Δ is the maximum price that a household with endowment (x, y) is willing to pay to switch to a house of better type x' . Thus $0 \leq \Delta(x'|x, y) \leq y$, and $u(x', y - \Delta(x'|x, y)) = u(x, y)$ unless $\Delta(x'|x, y) = y$. The proof of Lemma 1 rests on the following property, which is where normality enters.

Lemma A.1. *For all house types $x < x'$, the reservation price $\Delta(x'|x, y)$ is continuous and strictly increasing in y .*

Proof. The minimum in (A.1) is continuous in y , because u is continuous and $u(x', \cdot)$ is strictly increasing. Let $0 \leq y < y'$. If $y = 0$, then $\Delta(x'|x, 0) = 0 < \Delta(x'|x, y')$, the latter because $u(x', y') >$

$u(x, y')$. So let $y > 0$. For $s \in [x, x']$ and $z \in \{y, y'\}$, use $\tilde{y}(s|z) = \min\{t \geq 0 : u(s, t) \geq u(x, z)\}$ to denote the indifference curve that goes through (x, z) , truncated at zero, so that $\Delta(s|x, z) = z - \tilde{y}(s|z)$. It is continuous and nonincreasing in s , and wherever it is positive it satisfies $u(s, \tilde{y}(s|z)) = u(x, z)$ and, by the implicit function theorem, $\partial \tilde{y}(s|z)/\partial s = -MRS_{xy}(s, \tilde{y}(s|z))$. Since $u(x, y') > u(x, y)$, we have $\tilde{y}(s|y') > \tilde{y}(s|y)$ wherever the latter is positive. Let

$$D(s) = \Delta(s|x, y') - \Delta(s|x, y) = (y' - y) - [\tilde{y}(s|y') - \tilde{y}(s|y)],$$

so that $D(x) = 0$. On the interval where $\tilde{y}(s|y) > 0$, which contains a neighborhood of x because $\tilde{y}(x|y) = y > 0$,

$$D'(s) = MRS_{xy}(s, \tilde{y}(s|y')) - MRS_{xy}(s, \tilde{y}(s|y)) > 0$$

by normality, so D is strictly increasing there. On the rest of $[x, x']$, $\tilde{y}(s|y) = 0$ and $D(s) = y' - y - \tilde{y}(s|y')$ is nondecreasing. Hence $D(x') > 0$. $\square$

To prove Lemma 1, recall that a household with wealth w that faces price function p selects a house of type x with $p(x) \leq w$ to maximize $u(x, w - p(x))$, thus leaving $w - p(x)$ for consumption. In equilibrium, a household endowed with $\{\tilde{x}, \tilde{\theta}\}$ has wealth $\tilde{\theta} + p(\tilde{x})$, and every household is utility-maximizing at the equilibrium prices. So it suffices to show that, for any price function p , if two households with wealth $w_a < w_b$ choose optimal bundles $\{x_a, y_a\}$ and $\{x_b, y_b\}$, then $x_a \leq x_b$. Suppose instead that $x_a > x_b$, and let $P = p(x_a) - p(x_b)$. Household b can afford x_a , since $w_b - p(x_a) > w_a - p(x_a) = y_a \geq 0$; if $P < 0$ it would get a better house and more consumption, so $P \geq 0$, and its optimality requires $\Delta(x_a|x_b, y_b) \leq P$. Household a can afford x_b , where its consumption would be $w_a - p(x_b) = y_a + P < y_b$, and its optimality requires $\Delta(x_a|x_b, y_a + P) \geq P$. But then Lemma A.1 gives $P \leq \Delta(x_a|x_b, y_a + P) < \Delta(x_a|x_b, y_b) \leq P$, a contradiction.

The proof uses only utility maximization at given prices, so it applies equally to the discretized economy below and to the two-sided model of Section 3.4, where buyer wealth is just θ . The wealthier household has a larger budget set and attains a strictly higher utility, so PAM by wealth and house quality is equivalent with PAM by utility and house quality.

Proof: Existence of equilibrium Let's now discretize the house types, so that $0 < x_0 < \dots < x_N < \infty$. For brevity, we will refer to households endowed with a type- k house as households of type k . Denote the mass of type- k households with income equal to or lower than θ by $F_k(\theta)$. We assume that all of these conditional income distributions are continuous with full support $[\underline{\theta}, \bar{\theta}]$, where $0 < \underline{\theta} < \bar{\theta} < \infty$; that is, $F_k(\theta) = 0$ for $\theta \leq \underline{\theta}$, and F_k is strictly increasing on $[\underline{\theta}, \bar{\theta}]$. The mass of type- k households is $m_k > 0$, so $F_k(\bar{\theta}) = m_k$. Recall that we have normalized the total mass at $\sum m_k = 1$. Adding a constant to all prices $p = (p_0, \dots, p_N)$ does not affect any household's choices, so we normalize $p_0 = 0$. A type- j household with income θ has wealth $p_j + \theta$ and selects a house of type k with $p_k \leq p_j + \theta$ to maximize $u(x_k, p_j + \theta - p_k)$. An *equilibrium* of the discretized economy is a price vector p together with an assignment of households to house types such that almost every household is assigned a house type that maximizes its utility, and the mass of households assigned to each type k is m_k .

With discretized house types we require an additional assumption:

$$u(x_{k-1}, \bar{\theta}) > u(x_k, \underline{\theta}) \quad \text{for all } k = 1, \dots, N. \quad (\text{A.2})$$

This means that the distribution of autarky utility has overlap between neighboring household types; in other words, the grid of house qualities is sufficiently dense. We also require

$$u(x_{k+1}, 0) < u(x_k, \underline{\theta}) \quad \text{for all } k = 0, \dots, N-1, \quad (\text{A.3})$$

which means that no household would give up all of its consumption to trade up to the next house type. Like (A.2), it holds whenever the grid of house qualities is sufficiently dense; it holds for any grid when $u(x, 0)$ does not depend on x , as for CES utility (7) with $\rho \leq 0$.

Denote by $\gamma_k(y) = \Delta(x_{k+1}|x_k, y)$ the reservation price for trading up to the next house type of a household that consumes y in a type- k house.

Lemma A.2. *For $k < N$ and $y \geq \underline{\theta}$, $0 < \gamma_k(y) < y$ and $u(x_{k+1}, y - \gamma_k(y)) = u(x_k, y)$. Both $\gamma_k(y)$ and $y - \gamma_k(y)$ are continuous and strictly increasing in y , and $\bar{\theta} - \gamma_k(\bar{\theta}) > \underline{\theta}$.*

Proof. By (A.3), $u(x_{k+1}, 0) < u(x_k, \underline{\theta}) \leq u(x_k, y)$, so the minimum in (A.1) is positive, i.e., $\gamma_k(y) < y$, and $u(x_{k+1}, y - \gamma_k(y)) = u(x_k, y)$. Since $u(x_{k+1}, y) > u(x_k, y)$, the minimum is below y , i.e., $\gamma_k(y) > 0$. The minimum, $y - \gamma_k(y)$, is continuous and strictly increasing in y because $u(x_k, y)$ is, and $u(x_{k+1}, \cdot)$ is continuous and strictly increasing; Lemma A.1 gives the same for γ_k . Finally, by (A.2), $u(x_{k+1}, \bar{\theta} - \gamma_k(\bar{\theta})) = u(x_k, \bar{\theta}) > u(x_{k+1}, \underline{\theta})$. $\square$

The wealthiest household that consumes a type- k house must be indifferent between staying and trading up to the next house type. We therefore describe a candidate equilibrium by the upper bounds $\bar{y} = (\bar{y}_0, \dots, \bar{y}_{N-1}) \in Y = [\underline{\theta}, \bar{\theta}]^N$ of the consumption levels of households that consume each house type; by construction, $\bar{y}_N = \bar{\theta}$. The indifference conditions define the price increments $\Delta_k = p_k - p_{k-1}$ and the lower bounds $\underline{y}_k$ of the consumption intervals as functions of the upper bounds,

$$\Delta_{k+1} = \gamma_k(\bar{y}_k), \quad \underline{y}_{k+1} = \bar{y}_k - \Delta_{k+1} \quad \text{for all } k = 0, \dots, N-1, \quad (\text{A.4})$$

with $p_0 = 0$ and $p_k = \Delta_1 + \dots + \Delta_k$. Since γ_k is strictly increasing, a household that consumes y in a type- k house strictly prefers to trade up at the price Δ_{k+1} if $y > \bar{y}_k$, and strictly prefers not to, or cannot afford to, if $y < \bar{y}_k$. Type- j households with income θ will consume a type- k house if their wealth $p_j + \theta$ lies between $p_k + \underline{y}_k = p_{k-1} + \bar{y}_{k-1}$ and $p_k + \bar{y}_k$. Total demand for type- k houses is therefore

$$Q_k(\bar{y}) = \sum_{j=0}^N [F_j(\bar{y}_k + p_k - p_j) - F_j(\bar{y}_{k-1} + p_{k-1} - p_j)] \quad \text{for } k = 0, \dots, N, \quad (\text{A.5})$$

where, for $k = 0$, the second term is absent. Define the excess demand functions as $Z_k(\bar{y}) = Q_k(\bar{y}) - m_k$. Negative values of Q_k , which arise when the upper bound $\bar{y}_k$ is below the lower bound $\underline{y}_k$ implied by $\bar{y}_{k-1}$, are infeasible but harmless, and we do not remove them.

It is convenient to write demand in terms of wealth. Let $M(w) = \sum_{j=0}^N F_j(w - p_j)$ be the mass of households with wealth at most w , a continuous and nondecreasing function, and let $W_k = p_k + \bar{y}_k$ be the upper bound of wealth among households that consume a type- k house, with $W_{-1} = -\infty$ and $W_N = \infty$. Then $Q_k = M(W_k) - M(W_{k-1})$, and the cumulative excess demand for house types 0 to k , whose total mass is $\mu_k = m_0 + \dots + m_k$, is

$$\sum_{h=0}^k Z_h(\bar{y}) = M(W_k) - \mu_k.$$

For notational convenience, let $\mu_{-1} = 0$. Every household demands exactly one house, so $Z_N(\bar{y}) = -\sum_{k=0}^{N-1} Z_k(\bar{y})$, and all markets clear if $Z_k(\bar{y}) = 0$ for $k = 0, \dots, N-1$, or, in vector form, $Z(\bar{y}) = 0$. This is a system of N equations in N unknowns. The functions Z_k are continuous on Y , because the F_j and, by Lemma A.2, the γ_k are continuous.

Proposition A.1 (Existence). *The discretized economy has an equilibrium. In it, there are bounds of wealth $W_0 < W_1 < \dots < W_{N-1}$ such that households with wealth in (W_{k-1}, W_k) consume type- k houses, and the price increments satisfy $\Delta_{k+1} = \gamma_k(W_k - p_k)$ for $k < N$.*

Proof. Consider an imaginary “player” who adjusts the upper bound $\bar{y}_k$ in response to excess demand for type- k houses, within $[\underline{\theta}, \bar{\theta}]$:

$$a_k(\bar{y}) = \min\{\bar{\theta}, \max\{\underline{\theta}, \bar{y}_k - Z_k(\bar{y})\}\} \quad \text{for } k = 0, \dots, N-1.$$

(The top house type, where $\bar{y}_N = \bar{\theta}$ by construction, does not have a “player” representing it.) Hence $a(\bar{y}) = (a_0(\bar{y}), \dots, a_{N-1}(\bar{y}))$ defines a continuous vector-valued function from the compact convex set Y to itself. Thus, by the Brouwer fixed point theorem, there exists a fixed point $\bar{y}^* = a(\bar{y}^*)$. At the fixed point, for every $k < N$: if $Z_k(\bar{y}^*) < 0$ then $\bar{y}_k^* = \bar{\theta}$, and if $Z_k(\bar{y}^*) > 0$ then $\bar{y}_k^* = \underline{\theta}$.

Next we prove, by contradiction, that a fixed point must be a market equilibrium. Suppose that $Z_k(\bar{y}^*) \neq 0$ for some $k < N$, and consider the lowest such k , so that $M(W_k) - \mu_k = Z_k(\bar{y}^*)$. If $Z_k < 0$, then $\bar{y}_k^* = \bar{\theta}$: all households of types k or lower have wealth at most $W_k = p_k + \bar{\theta}$, and so do the type- $(k+1)$ households with income below $W_k - p_{k+1} = \bar{\theta} - \gamma_k(\bar{\theta}) > \underline{\theta}$, a positive mass by Lemma A.2; hence $M(W_k) > \mu_k$. Intuitively, no one among types k or lower is trading up to houses above k , while, due to overlap of autarky utilities, some type- $(k+1)$ households are trading down, so there must be excess demand for house types 0 to k . If $Z_k > 0$, then $\bar{y}_k^* = \underline{\theta}$: all households of types k or higher have wealth at least $p_k + \underline{\theta} = W_k$, so $M(W_k) \leq \mu_{k-1} < \mu_k$. Both cases are contradictions, so $Z_k(\bar{y}^*) = 0$ for all $k < N$, and then also $Z_N(\bar{y}^*) = 0$. Since $M(W_k) - M(W_{k-1}) = m_k > 0$ and M is nondecreasing, $W_0 < W_1 < \dots < W_{N-1}$.

It remains to show that households are maximizing utility. Assign the households with wealth in $(W_{k-1}, W_k]$ to type- k houses; this clears all markets, because M is continuous. Consider a household with wealth $w \in (W_{k-1}, W_k)$. It can afford a type- k house, because $p_0 = 0$ and, for $k \geq 1$, $w - p_k > W_{k-1} - p_k = \bar{y}_{k-1} - \gamma_{k-1}(\bar{y}_{k-1}) > 0$. For $k \leq l < N$, its consumption in a type- l house would be $w - p_l < \bar{y}_l$, so it strictly prefers x_l to x_{l+1} or cannot afford x_{l+1} ; since prices increase with house type, chaining these comparisons shows that x_k is strictly better than every affordable house type above it. For $l < k$, $w - p_l > \bar{y}_l$, so it strictly prefers x_{l+1} to x_l , and chaining shows that x_k is strictly better than every house type below it. Households with wealth exactly W_k have zero mass. Thus any fixed point of a indeed defines an equilibrium allocation of our model. $\square$

Equilibrium prices The next result shows that every equilibrium has the structure of the one constructed above. In particular, the equilibrium allocation of houses and consumption determines the price increments, so the prices that support a given equilibrium allocation are unique up to an additive constant. We do not show that the equilibrium allocation is unique.

Proposition A.2. *Suppose that p and an assignment of households to house types form an equilibrium, with $p_0 = 0$. Then there are bounds of wealth $W_0 < \dots < W_{N-1}$ such that almost every household with wealth in (W_{k-1}, W_k) consumes a type- k house, and the upper bounds $\bar{y}_k = W_k - p_k$ of consumption lie in $(\underline{\theta}, \bar{\theta})$ and solve $Z(\bar{y}) = 0$; in particular, $\Delta_{k+1} = \gamma_k(\bar{y}_k)$ for $k < N$. The consumption levels of households that consume a type- k house have full support on an interval $[\underline{y}_k, \bar{y}_k] \subseteq [\underline{\theta}, \bar{\theta}]$, where $\underline{y}_0 = \underline{\theta}$ and $\bar{y}_N = \bar{\theta}$: the unequivocally poorest and richest households do not have anyone to trade with.*

Proof. Let $\Delta_k = p_k - p_{k-1}$ and $M(w) = \sum_j F_j(w - p_j)$. First, $\Delta_k > 0$ for all k : otherwise a type- k house would be affordable whenever a type- $(k-1)$ house is, and strictly better, so no household would choose house type $k-1$, contradicting $m_{k-1} > 0$.

Second, households sort by wealth. For $k < N$, let W_k be the essential supremum of wealth among households that consume house types $0, \dots, k$, and W'_k the essential infimum among households that consume house types $k + 1, \dots, N$. By Lemma 1, $W_k \leq W'_k$: otherwise a positive mass of households that consume house types $\leq k$ would be wealthier than a positive mass of households that consume house types $> k$, and almost all of them are utility-maximizing. So almost every household with wealth below W'_k consumes a house type $\leq k$, and almost every household with wealth above W_k consumes a house type $> k$. Counting masses, and using the continuity of M , $\mu_k \leq M(W_k) \leq M(W'_k) \leq \mu_k$, so $M(W_k) = M(W'_k) = \mu_k$.

Third, $\Delta_k < \bar{\theta} - \underline{\theta}$ for all k . Suppose instead that $\Delta_k \geq \bar{\theta} - \underline{\theta}$ for some k . Then households of types below k have wealth at most $p_{k-1} + \bar{\theta} \leq p_k + \underline{\theta}$, and types k and above have wealth at least $p_k + \underline{\theta}$. Since $M(w) > \mu_{k-1}$ for $w > p_k + \underline{\theta}$, the sorting step gives $W_{k-1} \leq p_k + \underline{\theta}$; and since $M(p_k + \underline{\theta} + \varepsilon) < \mu_k$ for small $\varepsilon > 0$, $p_k + \underline{\theta} + \varepsilon < W_k \leq W'_k$. Hence the type- k households with income in $(\underline{\theta}, \underline{\theta} + \varepsilon)$, a positive mass, have wealth in (W_{k-1}, W'_k) , so almost all of them consume type- k houses, with consumption below $\underline{\theta} + \varepsilon$. But a type- $(k-1)$ house is affordable to them and, by (A.2), gives $u(x_{k-1}, \theta + \Delta_k) > u(x_{k-1}, \bar{\theta}) > u(x_k, \underline{\theta})$, which exceeds $u(x_k, \underline{\theta} + \varepsilon)$ for small ε by continuity, contradicting their optimality. Consequently M is strictly increasing on $[\underline{\theta}, p_N + \bar{\theta}]$, the support of the wealth distribution, which is the union of the overlapping intervals $[p_j + \underline{\theta}, p_j + \bar{\theta}]$. In particular $W_k = W'_k$, as both solve $M(W) = \mu_k$, and $W_0 < \dots < W_{N-1}$.

Fourth, let $\bar{y}_k = W_k - p_k$ for $k < N$. If $\bar{y}_k \leq \underline{\theta}$, then all households of types k and above have wealth at least W_k , so $M(W_k) \leq \mu_{k-1} < \mu_k$; hence $\bar{y}_k > \underline{\theta}$. Since M is strictly increasing at W_k , there are utility-maximizing households that consume a type- k house with wealth arbitrarily close to W_k from below, and others that consume a type- $(k+1)$ house with wealth arbitrarily close to W_k from above. The latter can afford x_{k+1} and weakly prefer it to x_k , so in the limit $W_k \geq p_{k+1}$ and $u(x_{k+1}, \bar{y}_k - \Delta_{k+1}) \geq u(x_k, \bar{y}_k)$. If $W_k = p_{k+1}$, this says $u(x_{k+1}, 0) \geq u(x_k, \bar{y}_k) > u(x_k, \underline{\theta})$, which contradicts (A.3). So $W_k > p_{k+1}$, the former households can afford x_{k+1} , and their optimality gives the reverse inequality in the limit. Thus $u(x_k, \bar{y}_k) = u(x_{k+1}, \bar{y}_k - \Delta_{k+1})$, i.e., $\Delta_{k+1} = \gamma_k(\bar{y}_k)$ by Lemma A.2. If $\bar{y}_k \geq \bar{\theta}$, then all households of types $\leq k$ have wealth at most W_k , and so do the type- $(k+1)$ households with income below $W_k - p_{k+1} = \bar{y}_k - \gamma_k(\bar{y}_k) \geq \bar{\theta} - \gamma_k(\bar{\theta}) > \underline{\theta}$, a positive mass; then $M(W_k) > \mu_k$, a contradiction. Hence $\bar{y}_k \in (\underline{\theta}, \bar{\theta})$. Since p is then the price vector that (A.4) assigns to $\bar{y}$, we get $Q_k(\bar{y}) = M(W_k) - M(W_{k-1}) = m_k$, i.e., $Z(\bar{y}) = 0$.

Finally, households that consume a type- k house have wealth filling (W_{k-1}, W_k) , so their consumption fills $(\underline{y}_k, \bar{y}_k)$ with $\underline{y}_k = W_{k-1} - p_k$. For $k \geq 1$, $\underline{y}_k > \underline{\theta}$: otherwise all households of types k and above would have wealth at least W_{k-1} , and $M(W_{k-1}) = \mu_{k-1}$ would require $F_{k-1}(\bar{y}_{k-1}) = m_{k-1}$, i.e., $\bar{y}_{k-1} \geq \bar{\theta}$. The lowest wealth is $p_0 + \underline{\theta}$ and the highest is $p_N + \bar{\theta}$, which gives $\underline{y}_0 = \underline{\theta}$ and $\bar{y}_N = \bar{\theta}$. (By the proof of Proposition A.1, every solution of $Z(\bar{y}) = 0$ in Y yields an equilibrium.) $\square$