Old problem set questions

The features explained in the initial part of a problem before any parts (a, b, c. . .) apply to the entire problem. The features in each part only apply to that part, unless otherwise noted. If there are further instructions below all the parts they also apply to the entire problem.

Read the whole question before starting to work on it. Sometimes it is a good idea to first solve a later part of a question, and refer to its solution in the answer to an earlier part. This can save time when the later part poses a more general version of the same question.

For a solution to be acceptable it must include the explanation behind your reasoning, including calculations where applicable. A mere bottom line answer is not acceptable.

Standard assumptions

These apply in all problems *unless otherwise stated*.

Decision-makers are risk neutral and maximize the present value of their own payoffs; this typically means profits for firms, and utility or consumer surplus for consumers.

When negative values would be nonsensical this is typically not pointed out explicitly but must be understood from the context. For example, a linear demand curve does not extend to infinity but only to the point beyond which the implied quantity or price would become negative.

For discounting purposes, assume that future periodic payoffs are realized at the end of a period. In particular, period $t = 0$ refers to now immediately ("current period", "this year", etc), period $t = 1$ to the end of first period ("one period from now", "next year", etc).

In discrete type pricing problems you can assume, for convenience, that a customer who is indifferent between two deals will choose the one that gives the seller more profit. No need to mess around with "minus one cent" prices to handle tie-breaking.

Even when a question asks you to find "the equilibrium" the correct answer might involve multiple equilibria or no equilibria. The singular phrasing is not giving away the answer of whether there is a unique equilibrium or not.

If the impact of a policy is not uniquely determined then it suffices to consider the best-case scenario for the decision maker (e.g., welfare or profits).

Sometimes it makes sense to use a computer for calculations, for example, Excel, Mathematica, or Python/NumPy. If you do, still make sure to explain your reasoning and show the equations you solved (but not your code).

Sections

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1 Competitive markets

1. Ann and Bob argue over the cause of a recent increase in the use of illicit drugs. Ann believes that television shows that glamorize drugs have induced more young people to start using drugs. Bob thinks that budget cuts in border security have made it easier to smuggle drugs into the country. Use a diagram to explain how and why the correct answer depends on whether the price of illicit drugs has increased or decreased. You can assume that the explanations offered by Ann and Bob are the only possible causes behind increased drug consumption.[1]

2. The demand for housing in Woebegon Heights is $Q^D(p) = 12000 - 3500p$, where the price is in thousands of €/m^2 and the quantity in thousands m^2. The housing stock in the Woebegon Heights is fixed in the short run and is currently 5 million m^2. Existing housing is supplied completely inelastically. Beyond the existing stock, the long-run supply of additional housing is $Q^S_+(p) = 1000(p-2)$ if price exceeds 2000 €/m^2 and zero otherwise.

   The Lake Woebegon region is badly hit by a pandemic. Many of the local services such as restaurants and theaters in the City of Woebegon have gone out of business. As a result, the willingness to pay for housing in the suburb of Woebegon Heights increases for some city residents, which causes the demand curve to rise to $Q^D(p) = 12000 - 2000p$.

   (a) What is the long-run equilibrium of the housing market in Woebegon Heights before the pandemic hits?

   (b) What is the short-run impact of the pandemic on the housing market in Woebegon Heights?

   (c) What is the long-run impact of the pandemic on the housing market in Woebegon Heights, assuming the change in demand is persists?

   (d) Continued from part 2c. In the very long run, as city services recover and the memory of the pandemic fades, the demand for housing in Woebegon Heights returns to its pre-pandemic level. What is the new market equilibrium?

   Illustrate your answers with graphs.

[1] This question is an age-old classic of elementary economics. Do not proceed further until you master this.

3. The demand for trinkets is $Q^D(p) = 100 - 2p$ pallets per week, where the price is in €/pallet.

   (a) The supply is $Q^S(p) = 4p - 20$. What is the market equilibrium (i.e., price and quantity)? How much consumer and producer surplus is generated?

   (b) There are 1000 suppliers, each with an individual supply of $q^S(p) = (p - 5)/250$. What is the market equilibrium? How much consumer and producer surplus is generated?

   (c) Continued from part 3b. Half of the suppliers improve their productivity, and as a result their individual supply curves are now $q_i^S(p) = (p - 5)/150$. What percentage of output is now produced by the higher-productivity firms?

A cubic meter of goodies (Midjourney).

4. The supply of goodies is $Q^S(p) = 15p - 6$ cubic meters per hour, where the price is in €/m^3. What is the price, quantity, total expenditure, consumer surplus, and producer surplus, in market equilibrium when...

   (a) The hourly demand is $Q^D(p) = 21 - 3p$.

(b) There are 300 buyers, each with an individual demand (in m^3/hour) of

$$q^D(p) = \frac{7}{50} - \frac{p}{100}.$$

(c) The demand comes from 500 buyers, each of whom will buy $0.1m^3$ of goodies if the price does not exceed their personal reservation price. This reservation price (€/m^3) is distributed uniformly between 0 and 10.

(d) The demand is 10 m^3/hour when price is 5 €/m^3, and there is a constant elasticity of demand $\varepsilon_d = -1.5$.

5. The nation of Druidia has been shocked by the doubling of electricity prices. A quarter of all electricity in Druidia used to be imported from neighboring Landia, but recently Landia banned all electricity exports due to local supply problems that had caused unrest over electricity prices.

   The electricity market in Druidia is organized as a double auction, where competitive buyers and sellers simultaneously submit their bids, and market price is determined as the equilibrium aka market-clearing price.

   (a) Provide a graphical illustration in the demand-and-supply framework of how the exit of Landian electricity exporters affected the Druidian electricity market. The graph can, of course, only be schematic as we have no data on demand and supply curves. However, the graph should be consistent with those quantitative features that can be inferred from the information that we have. Explain in your text what these features are and how your graph is consistent with them. As always, provide clear labels to all elements of your graph.

   (b) The political consultant and advisor to Druidia's government, Baldrick, devises what he calls a cunning plan to improve the lot of electricity consumers in Druidia. His plan is to replace the market mechanism with a new mechanism: after buyers and sellers have submitted their bids, the market-clearing price is calculated as before. However, now the price will be only 90% of the highest accepted seller price. What would be the likely impact of Baldrick's plan on the electricity market?

6. An unusually rainy Summer has resulted in a bumper crop of 32 tons of delicious porcini in Northland, causing its market price to plummet to 2.5 €/kg. Last year had a normal Summer, with total production amounting to 26 tons and price at 3 €/kg.

(a) Based on these two observations, give a back-of-the-envelope estimate for the demand elasticity of domestic porcini in Northland.

(b) Northland's new government is planning to cut down on working visas for seasonal mushroom hunters. Industry specialists anticipate this to reduce the porcini crop on a normal year to 22 tons. Using your previous estimate for the elasticity, give a back-of-the-envelope estimate for the impact of the new policy on porcini price and on total porcini revenue in a normal year.

7. The demand for solar panels is $Q^D(p) = 100 - p$ million units per year, where p is €/unit. The long run supply of solar panels is $Q^S_{LR}(p) = 2p - 20$. In the short run the supply depends on existing production capacity x: $P^S_{SR}(q) = P^S_{LR}(q) + 2(q - x)$. In long run equilibrium, x equals equilibrium output. (Here "capacity" refers to a normal level of output; even in the short run it is possible to produce more than x by using night shifts and other high-cost inputs.)

   Illustrate your answer to each part with a graph in supply-and-demand framework. You may illustrate multiple parts in one graph but make sure to explain what features refer to which part.

   (a) Initially the market for solar panels is both in short and long run equilibrium. Describe the market equilibrium (i.e., price and quantity). What is the equilibrium capacity x^*?

   (b) Continued from 7a. The price of carbon emission permits goes up, which leads to a doubling of the demand for solar panels. Solve for the new short run equilibrium.

   (c) Continued from 7b. In the long run, production capacity adjusts to the increase in demand. What is the new long run equilibrium?

8. The market for sanitizer has been stable for many years. The monthly demand has been $Q^D(p) = 25 - 0.25p$ and the supply $Q^S(p) = 0.2p - 2$, where the quantity is in barrels and the price in monetary units. Strange world events cause the demand to double, i.e., the quantity demanded goes up by a factor of two at every price.

   (a) What is the impact of the strange events on price, output and total revenue in the sanitizer market?

   (b) After the demand shock new firms begin to enter sanitizer production. Every month the supply increases by a further 20%. This expansion of supply only comes to an end once the market price is lower than the previous stable price. 12 months after

the strange events the demand suddenly returns to normal, but the new producers stay in the market.

Describe the evolution of price, output, and total revenue over time. You can assume that time is discrete and the market clears every month.

Numerous wind farms in a Finnish landscape, vintage poster (Midjourney).

9. The electricity market in Lintukoto has two types of producers. There are numerous wind farms that face a zero marginal cost of production. Their total output is 4000 MWh under normal conditions. However, depending on how windy it is, the output can be half or double the normal. There are also many other types of power plants, with a total capacity of 4000 MWh. The marginal cost of this capacity is uniformly distributed between 0 and 800 €/MWh.[2]

   The demand for electricity derives from 1000 buyers each with demand $q^d(p) = 10 - p/100$.

[2]As always, you can assume that producers have free disposal. Usually this goes without saying, but in reality electricity is one of the few goods where free disposal is not guaranteed and negative market prices are sometimes observed.

(a) What is the equilibrium market price (€/MWh) and hourly output (MWh) on a normal day? Illustrate with a graph that also points out consumer and producer surpluses.

(b) Suppose that half of the buyers have a fixed-price (FP) contract at which they face a constant price 400 €/MWh. The intermediaries that have sold these FP contracts are required to buy whatever amount the FP consumers demand. The other half of consumers face the market price of electricity.

Describe the market equilibrium on a high-wind day, normal day, and low-wind day. Illustrate with a graph.

(c) Continued from part 9b. Due to an external shock half of the non-wind power plant capacity exits the market. Unfortunately it is the half with the lower marginal costs, i.e., those between 0 and 400. Now what is the market equilibrium on a low-wind day, with and without the presence of FP contracts? Illustrate with a graph.

10. There are only two districts in Northland, Centria and Peripheria. The supply of housing in Centria is completely inelastic. In Peripheria, by contrast, supply is elastic due to the presence of many vacant but repairable houses. Households in Northland have a whole range of preferences over one district or another, described by how much more they are willing to pay to live in their preferred location. At first, both housing markets are in equilibrium, with price in Centria much higher than in Peripheria. Then a pandemic hits and the demand for housing goes up in both locations.

Use only few sentences and, in parts 10b and 10c, simple graphs to answer the following questions.

(a) What can be inferred about the distribution of location preferences in Northland?

(b) What is the impact of the pandemic on the housing market if there can be no movement between districts?

(c) What is the impact of the pandemic on the housing market if households can move between districts?

11. The bucolic town of Lintukoto is home to 15 identical lakeside summer cottages. The valuations (in k€) of their current owners are {44,47,55,65,77,85,91,101,122,128} and 5 other owners each have a valuation in excess of 150 k€. Summer is the trading season for cottages. This summer there are 9 other households with a positive valuation for a lakeside cottage. Their valuations are {35,50,63,74,80,82,95,110,130} (k€).

(a) How much total surplus can trading in this market generate? Use also a graph to illustrate this in the demand-supply framework.

(b) Suppose that there are in fact two markets for cottages, early summer and late summer, with potential buyers and sellers randomly divided between the two markets and without any knowledge of the other market. What kind of impact could this division have on total surplus and the number of trades, compared to 11a?[3]

(c) There are plenty of lakeside lots in Lintukoto, which are currently used for forestry but can be bought from local farmers at 20 k€ per lot. Suppose it is possible to quickly build new and equally attractive summer cottages on such lots at a construction cost of 40 k€. Now that there is potentially both trading and new construction in this market, how much surplus can it generate?

Excited buyer gets hold of a rare vinyl record. Magazine illustration (Midjourney). Excited seller gets a high price after selling a vinyl record. Magazine illustration (Midjourney).

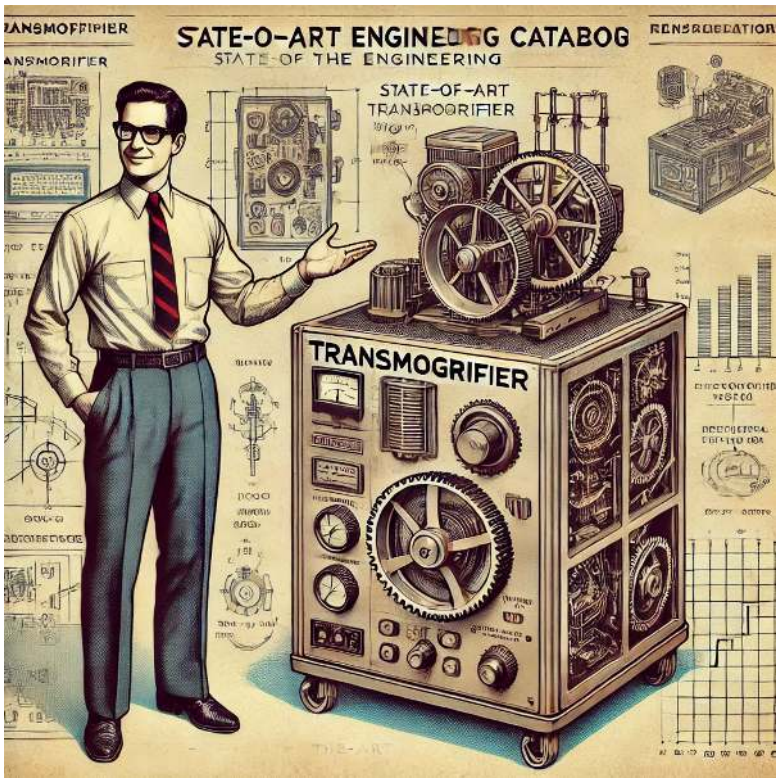
12. There are 6 people in the world who own an original copy of a particular vinyl record. Their valuations for the record are {40, 55, 65, 85, 135, 145} €. There are 7 other people with a strictly positive valuation for this record. Their valuations are {10, 30, 45, 65, 80, 85, 180} €. Potential buyers and sellers have been unaware of each other, until someone starts an online marketplace for used vinyl records, and then they all become aware of it.

[3] "Kind of" refers to a qualitative answer, such as sign or direction.

(a) How much total surplus would this new market generate in equilibrium?

(b) Suppose that, instead of all buyers and sellers entering the market at once, they arrive sequentially and trade without any knowledge of potential future trading partners. Could a fortuitous order of arrivals result in such a trading pattern that total surplus is higher than in 12a?

(c) Continued from 12a. How does the answer change if mailing a vinyl record from a seller to a buyer costs 10 €? Use also a graph to illustrate the impact of this transaction cost in the demand-supply framework.



A state-of-the-art transmogrifier presented by its proud new owner, in 1970s engineering product catalogue style. (DALL-E).

13. There are a thousand people in want of a transmogrifier. Their reservation values are distributed uniformly between 0 and 200 euros; no one would want a second transmogrifier. There are two thousand potential sellers, each in possession of one transmogrifier. Their reservation values are distributed uniformly between 100 and 200 euros.

(a) (6p) Illustrate the market equilibrium in demand-and-supply framework, with valuations on the vertical axis. What is the equilibrium price of transmogrifiers?

(b) What are the fractions of potential buyers and sellers who trade in market equilibrium?

(c) What is the elasticity of transmogrifier demand evaluated in market equilibrium?

(d) Continued from 13a. Instead of having to adhere to market equilibrium, suppose you can match potential buyers with potential sellers as you like. What is the largest amount of total surplus that could be generated?

(e) Continued from 13a. A mischievous middleman takes control of the transmogrifier market. His purpose is to maximize the number of trades. He can match potential buyers with potential sellers pairwise as he wishes, but trades must still be voluntary. Each participant can trade at most once. How many trades can the middleman implement? What is the resulting total surplus?

14. Use the NordPool day-ahead market data in file `finland-2021-12-07-H07CET.csv`, downloadable from MyCourses.[4] These are the actual demand and supply curves in Finland for one hour last December. The price P is in €/MWh and the quantities in GWh (= 1000s MWh).

    A note about how to deal with ties: due to the possibility of vertical and horizontal sections in demand and supply curves, their intersection is not guaranteed to define a unique equilibrium. If there is a range of prices at which demand and supply are equal, assume that the lowest price at which demand and supply equal is selected. If there is a range of quantities at which demand would equal supply then assume that the highest quantity is selected. If needed, the choice of traders from those with tied price offers would be randomized.

    (a) What was the equilibrium price, quantity, and total revenue? Illustrate with a graph.

    (b) There existed also a power station with 100 MWh of capacity that made no offer to sell electricity during this hour. Suppose it had offered to sell up to 100 MWh at 500 €/MWh. What would have been the impact on market equilibrium?

    (c) Continued from part 14b. Suppose that the same company that owns this idle power station owns also other power stations with marginal cost between 100 and 200 and a combined capacity of €/MWh 500 MWh. (This capacity was active and thus included in the supply curve). The marginal cost of using the additional capacity is

[4] `https://aalto-econ.fi/tervio/kandimikro/finland-2021-12-07-H07CET.csv`.
If you use Excel this is easiest to import using Data >> From Text/CSV.

400 €/MWh. There is also a one-time activation cost 10 k€ for turning on an idle power station.

What would have been the impact of activating the idle power station for one hour as was done in part 14b on the total profits of this company? You can assume that this power station is only turned on for one hour and is then idled again.

	A	B	C
1	P	demand	supply
2	0	11.9915	8.9319
3	1	11.9915	9.0111
4	2	11.9915	9.0111
5	3	11.9915	9.1861
6	4	11.7575	9.2336
7	5	11.7575	9.2811
8	6	11.7575	9.2811

A screenshot of the first few lines of the input `csv` for problem 14.

2 Market power

15. Tonttu Oy produces seasonal wreaths and is now contemplating its output and pricing decisions for the coming season. In most years the demand for its wreaths is $P^d(q) = 1800 - 30q$, where the price is in euros and q in tons. (This demand is the aggregate over many households, each with identical demand.) However, there is a 25% chance that the season turns out to be snowless, which would cause half of potential customers to spend the season elsewhere thus cutting the demand by half. Marginal cost is 300 €/ton and fixed costs are €6k per season.

    Describe price, output, and expected profits in the following scenarios.

    (a) The coming season is somehow known to be like the typical year for sure.

    (b) The coming season is somehow known to be a low-demand year for sure.

    (c) Output must be decided in advance, before the snow situation is known. Any unsold goods will waste before the next season. The producer must commit to a price in advance.

    (d) Continued from part 15c. Price commitment only prevents price hikes, but not price cuts.

16. The pharmaceutical company PanaceaGenix has invented and successfully trialed a new drug that treats early-onset ennui. The research and development phase incurred a cost of €1.5 billion, and the recently completed production facility cost another €500 million. The marginal cost of producing this drug is €2 per person-week of treatment. PanaceaGenix uses a yearly discount rate of 6%. There are 1 million potential customers, of whom 20% are rich and 80% are not. Among the rich, the reservation value for this treatment is uniformly distributed between 0 and 1000 €/year; among the poor it is uniformly distributed between 0 and 200 €/year.

    (a) What is the profit-maximizing simple price for a yearly dose?

    (b) Suppose it is possible to segment the market and charge higher prices from the rich than others. Describe the profit-maximizing pricing strategy. How much difference does the possibility to segment the market make to profits and consumer surplus compared to simple pricing?

Large pharmaceutical research lab, picture book illustration (Midjourney).

17. Arskan Kone is the only retailer of Acme's gadgets in the local market. It has 5 000 potential customers, each with a monthly unit demand for new gadgets. Their valuations are uniformly distributed in €$[5, 30]$. Arskan Kone can buy any amount of gadgets from Acme at €14 a piece. Arskan Kone has to pay a monthly rent of €10k for the additional floor area needed to sell Acme's gadgets, and also variable costs (such as sales clerk time) estimated at €2 per gadget sold.

    (a) What is the demand curve $P^d(q)$ for gadgets?

    (b) How does the profit-maximizing retail price depend on the monthly rent € $x \geq 0$? What are retailer profits and consumer surplus at $x = 10$k?

    (c) Suppose half of potential buyers only pay attention to the integer part of the price. That is, they make the purchase if the euro part of the price does not exceed their valuation (i.e., they ignore cents in prices). Now what is the profit-maximizing price?

18. Acme Inc is planning to produce disposable safety gear that helps protect against seasonal flu. The production would incur a fixed cost of €2 000, while the marginal cost is €10 for each pack of gear. The valuations of potential buyers are uniformly distributed in

€$[0, 20]$. One pack of gear lasts for the whole season, so no one would buy two at any price.

(a) There are 4 000 potential buyers. What is the demand curve $P^d(q)$ for Acme's gear?

(b) What is the profit-maximizing price when there are 4000 potential buyers?

(c) What is the profit-maximizing price when there are $N > 0$ potential buyers?

(d) Suppose there is uncertainty over whether the seasonal flu will in fact hit this season. If it does not then no one will buy Acme's gear at any price. (Unsold gear will expire before the next season). However, production takes time so Acme would have to produce its gear before finding out whether flu will actually hit this season. There are 4 000 potential buyers. How high should be the probability of flu hitting this season for the production of safety gear to be in expectation profitable for Acme?

Acme Inc disposable safety gear, product catalogue (Midjourney). Acme Inc disposable safety gear, centerfold of product catalogue (Midjourney).

19. On most years the sole producer of grumpkins in the island nation of Hy-Brasil faces a constant marginal cost of 300 pounds per lot and a fixed cost of 15 000 pounds. In most years the demand for grumpkins is $P^d(q) = 1800 - 60q$, where price is in pounds and q in lots.

(a) What is price, profits, and consumer surplus in most years?

(b) Marginal cost varies between years, it could be anywhere between 0 and 600 pounds. How do price, profits, consumer surplus vary with the marginal cost γ?

(c) Fixed cost varies between years, it could be anywhere between 10 000 and 20 000 pounds. How do price, profits, consumer surplus vary with the fixed cost ϕ?

(d) Demand for grumpkins varies between years, along with macroeconomic conditions in Hy-Brasil. This shows up in the choke price α, which varies between 1000 and 4000. That is, demand is $P^d(q) = \alpha - 60q$. How do the price and quantity of grumpkins vary with the demand shifter α?

Northland Transport bus line, 1940s style advertisement (Midjourney). Northland Transport bus line, 1970s style advertisement (Midjourney).

20. Northland Transport Inc is the only bus operator between the towns of Lintukoto and Tuonela. The fixed cost of operating this bus line is €7 200 per day. In addition, the marginal cost of each passenger is €20. The demand for daily bus trips between the two cities is $Q^d(p) = 200 - p$.

(a) What is the profit-maximizing price, the resulting profits and consumer surplus?

(b) The transport regulatory authority decides to impose a price cap on bus trips between these two cities. Which price cap would maximize expected consumer surplus?

(c) Suppose the regulatory authority was incorrect in its estimate of the fixed cost, which it used to set the price cap in part 20b. The actual fixed cost is €$(7200 + x)$, where $x \in [-7200, 7200]$. What is the impact of the error x on consumer surplus and profits?

21. The demand for electricity in the Lake Woebegon region is $Q^d(p) = 100 - 10p$. Quantity is measured in GWh per year and price in monetary units (MU) per GWh. The consumer price of electricity consists of the price of electricity, which can be bought in the competitive electricity market at 2 MU/GWh, and the transmission price. The transmission of electricity from sellers to buyers is provided by the PowerGrid Corporation. The annualized cost of maintaining a power grid with a transmission capacity of k GWh is $c(k) = 30 + k$ MU.

    What is the transmission capacity, consumer price of electricity, yearly consumer surplus, PowerGrid's profit, and total surplus, in each of the following cases?

    (a) PowerGrid Corp sets the price of transmission to maximize profits.

    (b) PowerGrid Corp sets the price of transmission to maximize total market surplus (aka "welfare"), subject to the constraint that it cannot make a loss.

    (c) The fixed costs of maintaining transmission capacity increase a little bit. What kind of impact does this have on the economically efficient amount of transmission capacity, and on the capacities chosen in cases 21a and 21b?

22. There are 1000 households in Busytown, each with a monthly demand for tap water of $Q_i^d(p) = 10 - p$, where the price is in marks per 1000 liters and the quantity in thousands of liters. The municipal waterworks faces a marginal cost of 1 mark for each $1000l$ of clean water produced. The maintenance of the waterworks infrastructure costs 3000 marks per month. The waterworks can only use uniform pricing.

    What is the price of tap water, monthly consumption per household, consumer surplus per household, total deadweight loss, and waterworks' profit in each of the following cases?

    (a) Waterworks maximizes its profits.

    (b) Waterworks maximizes consumer surplus.

    (c) Waterworks maximizes consumer surplus, subject to the constraint that it cannot make a loss.

Silkscreen print: Waterworks in Busytown (Midjourney).

(d) Half of Busytown's households move to Duckburg. How does this population decline affect the answer to parts 22a and 22c?

3 Public goods, welfare analysis

23. Housemates Hanna, Jaska and Kalle share a kitchen and a living room. Each of them hates cleaning work and evaluates the cost of cleaning work at €16/hour. Kalle has different preferences for cleanliness than the others. Measuring the units of cleanliness in terms of hours per week spent on cleaning the common areas, Kalle's demand for cleanliness is $P_K^d(q) = 24 - (2/3)q$. Hanna and Jaska have similar preferences, both valuing cleanliness at $P_H^d(q) = P_J^d(q) = 20 - q$. Cleanliness is never a bad for any of the housemates, but everyone has a point beyond which they feel additional cleanliness to be useless.

    (a) What is the efficient amount of cleaning in the common areas? What would be the surplus for each housemate, if the burden of cleaning is shared equally?

    (b) Suppose the housemates vote on the amount of weekly cleaning, the burden of which is to be shared equally. Under majority rule, what is the amount of cleaning and the resulting surpluses?

    (c) It becomes possible to outsource cleaning to a professional who charges €20/hour. The professional is so effective that one hour of cleaning by him is equivalent in output to two hours spent by any of the housemates. Now what is the efficient amount of cleaning?

Three housemates arguing about cleaning the kitchen, medieval manuscript illustration (Midjourney).

24. Researchers in Westland and Eastland are developing better treatments for lycanthropy, which afflicts no other region in the world. Inhabitants of both countries benefit from better treatments regardless of which country conducts the research. The country-specific marginal benefit from the research differs, however, due to differing prevalence of the disease as well as different trade-offs with respect to other potential uses of resources. In Westland the marginal benefit at Q researcher-years is $P_W^d(Q) = 96 - 12Q$ and in Eastland $P_E^d(Q) = 60 - 10Q$, where the benefit is measured in Large Monetary Units (LMU). The cost of a researcher-year is a constant 20 LMUs in both countries.

    (a) What is the efficient amount of research?

    (b) Suppose one of the countries jumps ahead and commits to provide exactly half of the efficient amount in part 24a. Taking this as given, how much will the other country then provide? Consider both cases of either Westland or Eastland being the first mover.

    (c) Suppose there is also a third country, Northland, that is worried about lycanthropy and has exactly the same preferences as Eastland. Now what is the efficient amount of research?

25. Residents of Northland fall into three clans, each with different valuations for Northland's common defense. The defense is provided by fighter planes, each of which costs 25 million euros. The aggregate valuation of each clan for the common defense is summarized by the following demand functions: $Q_A(p) = 60–6p$ for the Abelian clan, $Q_B(p) = 80–5p$ for the Babelian clan, and $Q_C(p) = 50–2p$ for the Cainian clan, where the quantity of defense is measured in the number of fighter planes and the value in millions of euros.

    Each clan is represented by a chief at national level. The chiefs' objective is to maximize the surplus of their own clan. According to Northland's constitution, the burden of defense spending is divided equally between the clans.

    (a) What is the efficient amount of defense spending for Northland?

    (b) If total spending is efficient, then what is the total surplus for each of the three clans?

    (c) The defense budget is decided by a majority vote in the council of clan chiefs. Each chief first proposes its most favored amount of spending, and then the chiefs vote on the proposals in the order determined by the chairman. How much would you expect Northland to spend on defense? How does the answer depend on which of the three chiefs holds the chairmanship?

26. The City of Boomtown is planning to build an underground service tunnel below Main Street. The cost of building the tunnel is 50 m€. There are 10 commercial properties on Main Street, each of which could be connected to the service tunnel at no additional cost to the City. Half of the properties have a high value for using the service tunnel, 8 m€; the other half each have a value of 3 m€. The city must treat all properties equally: it cannot charge different prices or taxes based on these connection values. All properties are operated by different real estate companies.

    Describe the resulting allocation and total surplus in the following cases.

    (a) The City will build the tunnel if it can recoup the construction costs from properties that pay to connect to the tunnel.

    (b) The City will extract as much revenue as it can from the joining properties; if that is not enough to pay for the tunnel then it covers the rest from a new property tax levied on all Main Street properties.

    (c) The City makes the efficient decision (the equal treatment constraint still applies!). If you find equally efficient choices then pick the one with lower taxation.

27. The Lintukoto Arts Foundation (LAF) runs the Lintukoto Museum of Contemporary Art. During summer months (from June to August) it has 1000 potential visitors per day. This is double the number of potential daily visitors during the rest of the year. The museum is open 30 days per month. The reservation values of potential visitors are always distributed uniformly in $[0, 10]$ €. The museum can accommodate up to 1200 visitors per day without becoming crowded. Each visit adds €2 to the costs, mainly due to security and maintenance. The objective of LAF is to maximize the total welfare generated by the museum.

    (a) What is the efficient number of daily visitors and the efficient price of a ticket in July and September respectively?

    (b) The museum also incurs a fixed cost of 270k €/year if it is to stay open to visitors. LAF has no other sources of funds so the museum must at least break even every year. Given this constraint, how should the tickets be priced in July and September respectively?

28. The planet Krikkit is suffering from carbon emission induced climate change. Emissions have now ended because the inhabitants burned up all their carbon deposits, but the sea level is expected to keep rising due to the stock of carbon in the atmosphere. The planet

consists of two countries. The marginal damage from rising sea level in Lowlandia is $X_1(s) = 20s$ K\$, where $s > 0$ is the rise in sea-level in Krikkitian feet (Kft). Highlandia begins to suffer damage only if sea level rises more than 10 Kft, in which case the marginal damage is $X_2(s) = 10(s-10)$ K\$. ("Marginal damage" is just a negative level of "marginal benefit".)

If nothing is done to remove carbon from the atmosphere the sea level will rise by 30 Kft. However, it is possible, but costly, to capture carbon from the atmosphere. Removing one Teraton of carbon costs 32 K\$. Each Teraton removed lowers the sea level by 0.1 Kft.

(a) Consider the reduction of sea level from the unmitigated level of +30 KfT as a good. What is the demand for sea level mitigation by Lowlandia and Highlandia respectively? Plot these demand curves with value (K\$) on vertical axes and sea-level mitigation (Kft) on horizontal axes.

(b) What is the efficient amount of carbon capture in Krikkit? Illustrate with a graph that plots the total benefit from sea level mitigation in Krikkit and the marginal cost of sea level mitigation.

Sea level rise is threatening the lowland inhabitants of Planet Krikkit! 1950s scifi magazine cover (DALL-E).

29. Municipalities in the Macondo metropolitan region share some common infrastructure. All costs are shared equally between the municipalities, each of which has one vote in any investment decision. There are three separate projects under consideration. Total cost and municipal gross benefits are depicted below in m€.

Project	(O) Opera (monumental architecture)	(S) Sewage treatment upgrade	(A) Airport to Downtown rail connection
Total cost	300	250	200
Macondo City	-5	20	120
Rosicruce	120	40	30
Uqbar	100	55	15
Tlön	50	65	70
Orbis Tertius	80	10	0

(a) Which investment projects would be efficient for the region?

(b) Suppose each investment decision is made one at a time. Which projects could get done under a majority rule? What about under a unanimity rule?

(c) Suppose it is possible to vote over combinations of projects. Find a combination that could pass a majority vote, even though it includes a project that could not pass if all projects are voted on separately. Is this "log-rolling" beneficial for economic efficiency?

30. All 100 owner-occupants of the housing company Hima Oy have one vote at the annual shareholders' meeting. One question is how much to spend on beautifying the courtyard. There are two types of owners, aesthetes (A) and busybodies (B), with busybodies being 3 times more numerous than aesthetes. Their individual marginal benefits from courtyard beautification are $P_A(q) = 0.2 - 0.01q$ and $P_B(q) = 0.1 - 0.02q$, where both valuations P and spending q are measured in thousands of euros per year. Neither type ever views beautification as a bad, but with sufficient spending both can become indifferent to further improvements.

(a) What is the efficient amount of spending on courtyard beautification at Hima Oy?

(b) All expenses at the housing company are shared equally between its shareholders, and spending decisions are made by majority vote. How much can we expect Hima Oy to spend on beautification?


Macondo City Opera House, monumental architecture brutalism, architectural model (Midjourney). Airport to Downtown rail connection in Orbis Tertius, Railway terminal, architectural model (Midjourney).

(c) There is a housing market with numerous housing companies inhabited by a large population of aesthetes and busybodies, all facing the same spending decisions as Hima Oy. Suppose the population were to self-select into housing companies by type, so that all owners would be of the same type as their neighbors. Compared with the majority decision in 30b, how would this affect the average surplus from courtyard beauty as enjoyed by individual aesthetes and busybodies respectively? It suffices to explain the direction of change for both types.

31. The electricity market in Trantor is in turmoil. The supply of fusion energy has dwindled due to problems with power station maintenance. The yearly supply of electricity is now $P^S(q) = 2q$, and the yearly demand is $P^D(q) = 200 - 0.5q$. Prices in Trantorian Ducats and quantities in TWh. The government reacts to the supply shock by eliminating the tax on electricity, currently at 40 TD/TWh.

(a) What are the price and welfare effects of this tax cut?

(b) In the first year the supply of electricity is completely inelastic, with 64 TWh supplied regardless of price. What is the short run impact of the tax cut on consumer price and government revenue?

(c) In the very long run the supply of electricity is much more elastic, namely $P^S_{LR}(q) = 80 + 0.75q$. What is the very long run impact of the tax cut on consumer price and yearly government revenue?

(d) Continued from part 31c. Use a graph to illustrate the very long run changes in consumer and producer surplus and deadweight loss the demand-and-supply framework. These changes are comparisons between what would have been the yearly surplus in the very long run with and without the tax.

Unauthorized tectonic activity (Midjourney). Market turmoil on Planet Trantor, scifi trading floor, frantic traders panic (Midjourney).

32. The electricity market in Trantor is once again in turmoil. The supply of geothermal energy has dwindled due to unauthorized tectonic activity. The yearly supply of electricity is now $P^S(q) = 40 + 2q$, and the yearly demand is $P^D(q) = 200 - 0.5q$. Prices are in Trantorian Ducats and quantities in TWh. The government reacts to the supply shock by imposing a price ceiling of $\bar{p} = 120$ TD/TWh.

(a) The electricity shortage that results from the price ceiling is resolved by optimal rolling blackouts. That is, the all-seeing regulators of Trantor are able to target the blackouts in such a way that their impact on total surplus is minimized. What are the welfare effects of the price ceiling?

(b) The electricity shortage that results from the price ceiling is resolved by rolling blackouts, the timing and location of which is governed by Murphy's Law. That is, the blackouts take place in the most inopportune moments and locales. What are the welfare effects of the price ceiling?

(c) Combining 32a and 32b. Use a graph to illustrate the deadweight loss in both the best-case and the worst-case scenarios in the demand-and-supply framework.

(d) In the long run all electricity suppliers need to maintain and renew their productive capacity, and so some producers that are willing to supply electricity at $\bar{p}$ for this year will eventually exit the market if the price ceiling persists. More precisely, in the long run the yearly supply of electricity is $P^S_{LR}(q) = 104 + q$. What is the long run impact of the price ceiling on consumer surplus in the best-case scenario?

Nikod II, the despot of Molvania, aggressive propaganda poster (Midjourney).

33. The demand for cabbage in Molvanîa is $P^d(q) = 100–30q$ while the supply is $P^s(q) = 5 + 15q$, where prices are in Strubl per kg and quantities in tons per year.[5]

[5]Useless information on Molvanîa available here: http://www.jetlagtravel.com/molvania/useful.html.

(a) Cabbages have historically been taxed at $t = 30$ (Strubl/kg). What are the welfare effects of this tax in Strubl/year? Illustrate with a graph.

(b) The local despot, Nikod II, is only interested in maximizing the tax revenue from cabbage. Illustrate the relation of tax revenue and the tax level t. How much could Nikod II extract?

(c) The despot is displaced in a coup d'etat. A new reformist government attempts to maximize the welfare of Molvanîan citizenry, including the consumers and producers of cabbage. It vows to use all cabbage tax revenue to produce public goods. If the citizenry values public goods that cost 1 Strubl to produce at 1.5 Strubl then what is the optimal level of the cabbage tax?

34. Illustrate the following case in demand-and-supply framework. The nation of Stetsonia has recently instituted a milk subsidy of 8 €/liter. Total consumption of milk is now 6 million liters per year, while the producer price is 16 €/liter.

(a) Highlight the regions that represent the following euro-valued expressions. i. Deadweight loss from the subsidy, ii. Consumer surplus, iii. Producer surplus, iv. Government expenditure.

(b) In the long run, the supply of milk responds to the subsidy. The long run supply curve has the same choke price as the short run curve seen in part 34a. Demand curve stays unchanged, but total consumption grows to 8 million liters per year. Highlight the regions that represent the following euro-valued expressions. i. Difference in long and short run producer surplus, ii. Difference in long and short run deadweight loss. Use different colors for positive and negative contributions to the difference.

Use linear demand and supply curves with positive choke prices. There may be many combinations of demand and supply that satisfy the required criteria. If some highlighted regions become unclear due to overlap, then use separate graphs for clarity, but the assumed demand and supply curves have to be the same in every graph.

35. The yearly demand for horse milk in Berserkistan is $Q^D(p) = 20 - 0.05p$ while the supply is $Q^S(p) = 0.2p - 40$, where prices are in dollars and quantities in kilotons.

(a) The government of Berserkistan has long been beholden to its powerful horse milk producers, and has been paying them a subsidy of \$100 for each kiloton (kt) produced. What are the welfare effects of this subsidy?

(b) To quell a revolt by urban taxpayers the government ends the production subsidy for horse milk. Instead the government begins to pay the buyers of horse milk a consumption subsidy of 100 \$/kt. What are the welfare effects of this subsidy?

(c) Berserkistan transitions into a two-party democracy. Half the time the power is held by the Farmers' party and the other half it is held by the Urban party. In years when the Farmers' party is in power the production subsidy is raised to 200 \$/kt, whereas when the Urban party is in power it is not paid at all. What are the welfare effects of this unstable policy on average across the years?

(d) A coalition government comes to power in Berserkistan. A grand bargain is struck between farmers and urban dwellers: all subsidies are eliminated, and producers are promised the same level of welfare as they used to have when they received the 100 \$/kt subsidy (in part 35a). The government achieves this by buying horse milk from the market and then selling its purchases in the world market at a price 40 \$/kt. Importation of horse milk is banned in Berserkistan. What are the welfare effects of this new policy?

Election poster, Farmer's Party of Berserkistan (Midjourney). Election poster, Urban Party of Berserkistan (Midjourney).

36. Nation A gets all of its tax revenue from taxing land and employers. The supply of labor is $P^s(q) = 2q$ and the demand is $P^d(q) = 200 - 0.5q$, where q is in millions of labor-years and prices in thousands of monetary units (kMUs). To keep things simple, nation A has a

flat unit tax on employers at 60 kMU per every labor-year. Any changes in land taxation or existing public spending are unthinkable due to political reasons.

(a) How much revenue does Nation A raise with the tax on employers?

(b) Describe the incidence of this tax in terms of revenue shares (% of total employment tax revenue).

(c) Legislators in Nation A are considering the provision of a new public good, which is estimated to have a total benefit twice as high as its cost. This new good would be a relatively small expense, in that its total cost is very small relative to current total tax revenue. Potential sellers are abroad, so this would have no direct impact on the local labor market, but it would necessitate a small increase in the employer tax revenue. Would this public good provision increase total surplus in Nation A?

37. There are two million households in the country of Hyperborea, and each will always live in one house. They face the choice of whether to live in the city or in the periphery; household valuations for living in the city are distributed uniformly between -1000 and 1000 Talon/year. (A negative valuation captures a preference for living in the periphery.) All households have an income above 2000 T/y. The are 600k houses in the city, and it is illegal to build more. Houses in the periphery are not scarce. Only occupied houses incur a maintenance cost of 200 T/y. Temperature in the city varies between -30°C and 30°C. Everyone uses a discount rate of 5%.

(a) Households rent their homes for one year at a time. Houses are owned by competitive landlords who pay for the maintenance costs. The government is considering a property tax on occupied houses at τ Talon/year. Consider a low tax $\tau_L = 100$ and a high tax $\tau_H = 500$. What would be the impact of the tax on where Hyperboreans live? How much revenue and how much deadweight loss would it generate?[6]

(b) As in part 37a, but the tax is only applied in the city.

(c) What is the impact of the taxes in part 37b on house prices? The market price of a house is the present value of rents net of taxes and maintenance costs.

[6]Hint: consider demand and supply of housing in the city, treat what happens in the periphery as the leftover.

4 Tools, foundations

38. Acme Ltd produces two types of gadgets, A and B. Their production requires specialized equipment which it leases from another company at 500k euros per year. The same equipment can be used for both gadget types, and year is the shortest possible rental period. The marginal cost of both types of gadgets is 100 euros a piece.

    (a) If Acme is facing only a single order of 1000 gadgets of type A, then what should be Acme's reservation price per gadget?

    (b) Suppose Acme expects that there is a 50% chance that it will get another order later during the year, and that this order would be for 2000 type B gadgets at a take-it-or-leave-it price of 200 euros a piece. How would this affect the answer to part 38a?

    (c) Acme has a production capacity of 1000 gadgets of type A and 2000 gadgets of type B. It is a price taker in the world gadget markets. At which combinations of world market prices $\{p_A, p_B\}$ should Acme produce both types of gadgets? Or A only, or B only?

39. Lämpö Oy is a small HVAC company that installs heat pumps to private homes. It provides a full service that covers both the heat pump and the installation. Lämpö is a small player and takes the price of its service as given, so it just charges what appears to be the "going rate" in the market at the time. It just decides how many customers to take. Lämpö has made a deal with a heat pump manufacturer that allows it to buy up to 20 units per week at a wholesale price of 1000 €/unit. Beyond that the pumps would have to be bought from the open market at a cost of 1500 €/unit. Lämpö has four employees. It takes two employee-days to install one heat pump. Lämpö is currently paying a wage of 150 €/day for working from Monday through Friday. It is possible to get the workers to also work on Saturdays at 250 €/day. Two of the four installers would also be willing to work on Sundays at 400 €/day. All other costs of running the business are sunk for the time being.

    (a) What is the cost function of Lämpö Oy? Draw average and marginal costs (€/unit) in the same graph with the number of installations (units/week) on horizontal axes.

    (b) Suppose the going rate for installations is 1600 €/unit. What level of production would maximize profits? What would be the profits and the total earnings of its employees?

(c) How does Lämpö's output depend on the price of installations, i.e., what is its supply curve?

Strawberry farm, army of sweating berry pickers working hard, manga drawing (Midjourney).

40. A berry farm produces strawberries for the mass market. It expects to sell any amount of berries at a market price which it has no influence over. It is able to produce up to 80 tons (1000s kg) of strawberries this year. All other costs have been sunk, but the berries have yet to be picked. The harvest season is short, and picking each and every berry before they go bad is harder then just picking the easiest-to-reach berries. Half of the berries could be picked at a rate of 10 kg/worker-hour. Another quarter of the berries could be picked at a rate of 5 kg/worker-hour, while the hardest-to-reach quarter can only be picked at a rate of 3 kg/worker-hour. The wage rate of pickers is 10 €/hour. The farm can hire pickers after finding out this season's strawberry price level.

(a) What is the cost function of the farm? Draw average and marginal costs in the same graph.

(b) Suppose the price of strawberries is 1200 €/ton. What level of production would maximize profits? What would be the profits and the total earnings of the pickers?

(c) Suppose the wage rate of pickers were higher, or lower, by 2.50 €/hour. How would this impact profits and pickers' total earnings?

41. There are two methods for producing the same quality of lighting for one unit of office space: using LEDs or using incandescent light bulbs. LEDs are more expensive but need replacement less often. The problem is to figure out the least costly course of action for producing the desired level of lighting.

Type	Power (W)	Durability (h)	Price (€)
LED	1	50 000	24
Bulb	50	2000	1

Price of electricity is 0.10 €/kWh, and the opportunity cost of capital is a 4 % return. Usage is 2000 h/year (about 5.5h/day). Electricity bill is paid at the end of each year. A device with power x W uses x kWh of electricity per each 1000 hours of use.

(a) Compare the cost of producing light between these two methods over the next 25 years. Which method would be more cost-efficient?

(b) Suppose the price of LEDs is expected to drop by 50% every year. When should the first LEDs be purchased?

42. Wally has come up with a cost-saving idea, described in the following document: https://www.accum.se/~zqad/webcom3/dilbert/2009/090111.gif

(a) What is the present value of Wally's idea as a function of the discount rate $r > 0$? What if $r = 0.03$?

(b) What is the smallest integer number of years T, at which it holds true that even if Wally's idea only delivers its benefit for the first T years, it still provides at least 99% of the present value of an infinitely lasting idea? Assume a discount rate of 3%.

43. Dutch settlers purchased the island of Manhattan from the Lenape tribe in 1626 at a price of 60 Guilders, which amounted to 0.6 kg of silver. The current market value of silver is about 500 USD/kg. Supose the Lenape had invested all the proceeds on an interest-bearing silver account.

(a) What would be the current value of the Lenape silver holdings, had the silver account returned a constant rate 2.5% per year? What if the rate was 5% per year?

(b) The current market value of all real estate on the island of Manhattan is about 1 trillion (10^{12}) USD. How high should a constant rate of return on the Lenape silver deposit had to have been for their wealth to now equal this value?

44. The product line of Penny Ltd. includes a novelty item, which is very likely to return a tidy profit of 0.1 m€ each year. However, every year there is a 1/100 chance that the novelty item malfunctions. In a year when that happens the company has to pay various damages and legal costs, and the novelty item makes a loss of 1 m€.

    Steamroller Inc. is considering a purchase of the rights to this novelty item from Penny Ltd. The transfer of ownership would not change the probability distribution of the profits it generates. Steamroller uses a yearly discount rate 5%. What should be its reservation price for the novelty item in the following cases?[7] This situation repeats...

    (a) every year forever.

    (b) every year for the next 40 years, and then the novelty item is retired.

    (c) until there is a malfunction, and then the novelty item is retired.

45. Mephisto Inc is considering a cost-cutting decision that would increase its profits for the next T years by one billion euros per year, starting this year. However, these cost savings would come back to bite and forever decrease Mephisto's profits by one billion euros per year starting in year $T+1$. Mephisto's discount rate is 5%. For which (integers) T should Mephisto choose to do this cost-cutting? Why and how does the discount rate impact Mephisto's choice?

46. A team of researchers has obtained a permit from Statistics Finland (TK) to use administrative data sets in their research project. For confidentiality reasons the data can only be used at a computer at TK. The team's objective is to minimize their costs while implementing their project. The team faces a trade-off: the straightforward way of implementing the research requires little research assistant time but results in a high peak load of RAM. The project can only be implemented if the peak-load does not exceed the actual RAM available. However, the RAM requirement can be reduced by spending more

[7] You can assume that the purchase price would be paid immediately, and the buyer would get the profits starting next year.

CEO of Mephisto Inc, cartoon character, BW drawing (Midjourney).

research assistant hours on menial code optimization. The resulting peak-load m (in GB) as a function of assistant hours h is $m(h) = \max\{60 - 6\sqrt{h}, 10\}$.

(a) The project will last 3 years for which TK charges the following prices depending on the amount of RAM: 16GB is priced at €8250, 32GB at €10500, and 64GB at €14400.[8] Research assistants can be hired at €15/hour. What is the cost-minimizing way of implementing the project?

(b) Continued from part 46a. The team convinces new management at TK to give researchers the option of bringing their own computer to do their computing inside TK. This would involve paying a market price of €1000 for the computer excluding the RAM, which adds to the price depending on the capacity: 16GB at €100, 32GB at €200, or 64GB at €400. Now what is the cost-minimizing way of implementing the project?

[8]Pricing accurate as of September 2021:
https://www.stat.fi/tup/mikroaineistot/aineistojen-ja-palveluiden-hinnat_en.html

(c) Suppose any amount of computing time with 64GB of RAM can be rented from TK at a cost of €20 per Convenient Time Unit (CTU). The computing time required by the project happens to be linearly related to the peak memory requirement, so that $m(h)$ is also the computing time requirement in (CTU) if h hours is devoted to menial code optimization. What is the cost-minimizing decision as a function of the price of research assistant time c €/hour? What if $c = 15$?

47. There are 1 million households in Lintukoto and 250k apartments in its core region. The rest of Lintukoto is the periphery, where land is not scarce and rents are determined by costs. It always costs 10 k€/year to rent a house in the periphery. All households have a reservation value for living in the core instead of the periphery; it is uniformly distributed in $[-50, 50]$ k€/year. This value takes into account both location preferences and the higher wages that can be earned in the core region. (The reservation value is negative for those who strictly prefer to live in the periphery.) In Lintukoto everyone can borrow and lend at a 5% interest rate. Assume all rents are paid at the end of the year.

(a) Suppose all households are renters. What is the equilibrium rent of a house in the core region (in k€/year)?

(b) The equilibrium price of a house in the core region is equal to the present value of rents that it generates. What is the equilibrium price?

(c) Labor productivity and thus wages are expected to rise faster in the core than in the periphery. Suppose the resulting change is demand is known to take the form of a permanent increase in every household's reservation value by 20 k€/year in exactly 10 years from now. How would the rent have to change over time for the market to be in equilibrium every period?

(d) Continued from 47c. How many houses would have to be built in the core region to keep the rents stable at their initial level?

(e) Continued from 47c. How would the price of houses have to change over time to be consistent with this development? Here it suffices to draw a rough sketch of the time series for the price for the next 15 years, with exact values pointed out at $t = 0$ and $t = 10$. Qualitatively, how does the price-to-rent ratio change over time (i.e., when is it decreasing/increasing/constant)?

48. Every year, a producer of balsamic vinegar prepares a new vintage and stores it in wooden barrels for aging. During the first year of aging the sale value of vinegar increases by 15%. In every subsequent year, the rate of value increase is reduced by one percentage point.

For example, in the third year the value increases by 13%, in the fifth year by 11%, and so on, until eventually the value begins to decrease. The producer's discount rate is 4.5%. The initial value of a new vintage is 40 €/l.

(a) What is the present value of a new vintage, in €/l, if the producer plans to sell it 5 years from now?

(b) What is the optimal duration for the aging process?

Screen capture from analogue television: game show True or False! Gleeful host, apprehensive contestants Ukko and Akka facing off for the main prize of €20,480. (DALLE).

49. The most popular game show in Lintukoto consists of a series of true-or-false questions. The prize for getting the first answer right is €10. After getting an answer right the contestant can either quit and leave with the current prize money, or enter another round where the total prize money for the night is doubled. However, if at any point a contestant gives an incorrect answer then they will go home with zero prize money. The maximum number of questions for a contestant is 12, after which the show would end.

(a) Give a mathematical description of the "lottery" implied by the game show, for a contestant who never voluntarily quits and whose probability of getting the right answer is the same $p \in (0, 1)$ for any question.

(b) Ukko has starting wealth €100,000 and a utility function over wealth $u(w) = \sqrt{w}$. He is clueless about the questions and believes that his probability of getting any single answer correct is 50%. If he were planning to quit after two questions then what would be Ukko's certainty equivalent for the game? What if he were planning to never quit?

(c) Continued from 49b. After how many correctly answered questions should Ukko choose to quit the game?

(d) Akka has the same wealth and preferences as Ukko in part 49b, but a 75% chance of answering any single answer correctly. What should be her reservation value to get to play the game?

Ann and Bob pensively pondering, facing risky investment choice, bw photography (Midjourney).

50. Ann and Bob have access to a risky investment opportunity, which has an equal chance of yielding either a 20% positive return or a 10% loss. Their risk aversion is well described by constant relative risk aversion (CRRA), but with different levels of relative risk aversion: $\rho_A = 0.5$ for Ann, $\rho_B = 1$ for Bob, and $\rho_C = 0$ for Cecilia. Each investor has to stake their entire wealth to take part in this investment. The alternative is to get a safe return R% on the entire wealth. What are their reservation values for the safe return R needed for them to accept the safe option instead of the gamble?

(a) When they have initial wealth levels of €1 million.

(b) When they have initial wealth levels of €10 million.

51. A punter with inside information knows that a sports bet that others think has a 50-50 chance of paying out has, in fact, a 2/3 chance of paying out. Every €1 wagered on this particular bet turns into €2 in case of success and is lost in case of failure. The punter's preferences over wealth are described by the logarithmic utility function $u(w) = \log(w)$, where $w > 0$ is in €m.

    (a) The punter's initial wealth is €12m. It is possible to wager only either 1, 4 or 8 €m on this bet. Among these risky choices what is the highest certainty equivalent the punter can attain?

    (b) As in 51a, but diminish initial wealth and the possible amounts wagered by a factor of ten: wealth is now €1.2m and wagers either 100, 400 or 800 €k.

    (c) Any amount can be wagered. The punter decides which fraction of initial wealth to invest in this gamble. What is the optimal fraction and how does it depend on initial wealth w?

52. A cloud computing company offers various deals to its customers. It sells data processing services at a constant price 2 €/TPUh. It also offers a package of 100 TPUh at a price of €50 and a package of 200 TPUh at €80. A customer may buy any number or combination these deals. Any unused paid-for TPU hours expire after a week.

    (a) Describe the cost function for data processing for one customer, who might use anything between 0 and 300 TPUh in a week. Plot the cost function on a graph.

    (b) Describe the budget set of a buyer, who is allocating a weekly computing budget of €150 between cloud services and equipment, but will never buy more than 300 TPUh/week. Equipment is measured in units that have user cost €1/week and can be rented in any continuous amount. Plot this budget set in consumption space with equipment (in euro-equivalent units) on horizontal and cloud processing (in TPUh) on vertical axes.

    (c) While there is much variety in their exact shapes, all potential customers have "smooth preferences" over these two goods. That is, their indifference curves are smoothly curved in consumption space. Some particular levels of weekly TPUh consumption are observed to be more common than other nearby levels. What would you expect these common consumption levels to be, and why?

53. Anna is a typical consumer and taxpayer in Landia. She consumes electricity (E) and a basket of other goods (B). Her budget is initially €1200 but income tax takes away 25% of it before she spends the rest on private goods. (This income tax revenue is used to produce a public good that does not affect Anna's utility from private goods, nor does she adjust her working in response to income taxation.)

    Anna's preferences are nice and smooth: her indifference curves have no linear sections nor any kinks. She would always prefer more to less, other things equal. If offered two hypothetical consumption baskets that she is indifferent with, she would strictly prefer an average of the two to either of them. Anna neither saves nor borrows.

    Last period market prices were 10 €/unit for electricity and 20 €/basket for other goods. However, this period the market price of electricity triples.

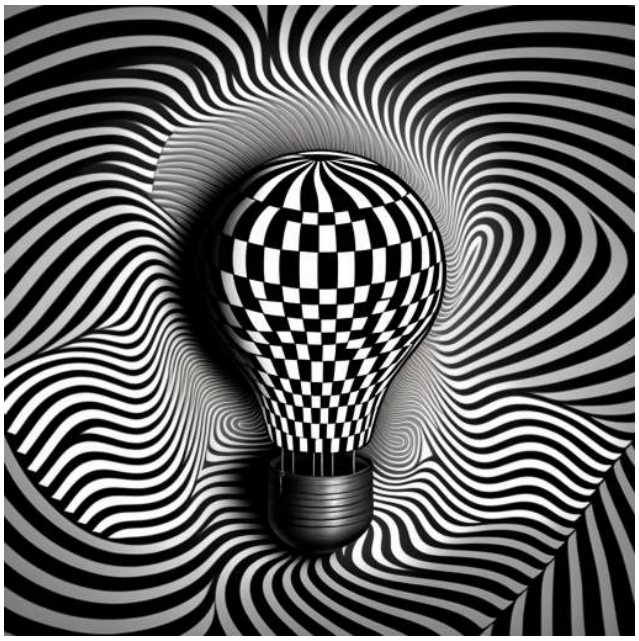
    (a) What is the equation for Anna's budget curve before and after the price change?

    (b) Last period Anna spent a third of her budget on electricity and the rest on other goods. After the price change she plans to spend half of her budget on electricity. Draw a graph of Anna's consumption space with electricity on vertical axes, and depict her budget sets and consumption bundles before and after the price change. Draw also two indifference curves that are consistent with her choices and represent her utility before and after.

    (c) Continued from 53b. How much does Anna's spending on electricity go up? How much money would she have to be given to be equally well off as before? The latter answer depends on what kind of indifference graphs you gave Anna in part 53b. Use that graph to show how you obtained the answer.

    (d) Under pressure from unhappy consumers the government in Landia decides on new electricity subsidies to restore the consumer price of electricity to its previous level. To fund this spending the income tax is increased. For Anna, being both an average consumer and an average taxpayer, this means that her level of disposable income is reduced by exactly the same amount that the government ends up spending on her electricity consumption.

        Does this policy make Anna better off, worse off, or make no difference to her? The answer does not depend on what kind of indifference curves you drew for her. Justify your answer with the help of a graph of Anna's consumption space.



Anna the consumer-taxpayer, self-portrait (Midjourney). Op-art depiction of electricity (Midjourney).

54. Consumers in the nation of San Escobar consume gasoline (y) and other goods (x). Their preferences over yearly consumption are represented by $u(x, y) = x^{0.75}y^{0.25}$. Consider a representative consumer with a budget of $M = 10$ Pbs. The market prices are currently $p_x = 1$ and $p_y = 4$ Pbs/unit.[9]

    (a) How much does the representative consumer consume of each good in a year? Draw the budget set and (at least) one indifference curve for the representative consumer, with x on horizontal axis. One indifference curve should go through the optimal consumption bundle.

    (b) The current market price of gasoline is twice as high compared to last year, due to a supply shock in the world oil market. Motivated by a history of gas price riots, the government of San Escobar decides to subsidize gasoline by 2 Pbs/unit to bring its price to local consumers back to where it was before the oil shock. However, the government has no other sources of income besides the local population. It pays for the subsidy by imposing a tax on other goods at τ Pbs/unit.

    What is the impact of this policy on consumer welfare? (It suffices to explain the direction of change). Now how much do consumers spend on each good in a year?

[9]The geographic map of San Escobar does not provide any clues to answering this question. https://www.defactoborders.org/media/pages/places/san-escobar/gallery/5ab751c4c5-1620049648/img.jpg

55. A consumer has the utility function $u(a,b) = a^{\frac{1}{4}}b^{\frac{3}{4}}$, where a is apples and b the baskets of all other goods. The internal structure of the basket b is not affected by the levels of consumption. The price of b is normalized at $p_b = 1$, in other words, it is the *numeraire* in terms of which other prices are measured. The price of apples is simply denoted by p.

    (a) If the consumer has a budget of $M = 100$ and $p = 0.5$, then which consumption bundle would maximize her utility?

    (b) In general, if this consumer has a budget of $M > 0$ and the price of apples is $p > 0$ then what is her (Marshallian) demand for apples?

    (c) What is the expenditure share of apples in this consumer's total spending?

56. In Lintukoto everyone buys exactly one new phone every 2 years. In the old days life was simple and only BrickPhones were available, priced at €100. Then one year also FancyPhones become available, priced at €300. Everyone in Lintukoto agrees that a FancyPhone is better than a BrickPhone. In the first year half of buyers choose a BrickPhone. After that, in successive years, FancyPhone increases its market share by 10 percentage points until BrickPhones have no buyers left; a year after BrickPhone market share reaches zero it is no longer sold in Lintukoto.

    Let's denote the last year before FancyPhones become available as "year 0" (zero).

    (a) What is the average price of a phone sold in Lintukoto in years $0, 1, \ldots, 6$?

    (b) What is the average price of a phone model available in Lintukoto in years $0, 1, \ldots, 6$?

    (c) Economists in Lintukoto are trying to measure changes in real incomes. To do this they have to measure changes in the price level of cell phones, as one of the many components of the overall price level. Explain in a couple of sentences what can be inferred about the contribution of cell phones prices to the overall price level.

57. A firm that produces stuff is a price-taker in the world market for stuff. It has a fixed cost of 10 monetary units (MUs) and a marginal cost of $2 + 0.2q$ MUs.

    (a) What is the firm's supply curve?

    (b) What are the firm's profits as a function of p, the price of stuff.

(c) The world is full of potential producers that could produce stuff with this same cost structure. More of them will enter the market if profits are available, while some will exit if it is not possible to earn at least zero profits. In equilibrium, all firms make zero profits and no firm can gain by increasing or decreasing its level of output. How much stuff does each firm produce in equilibrium?

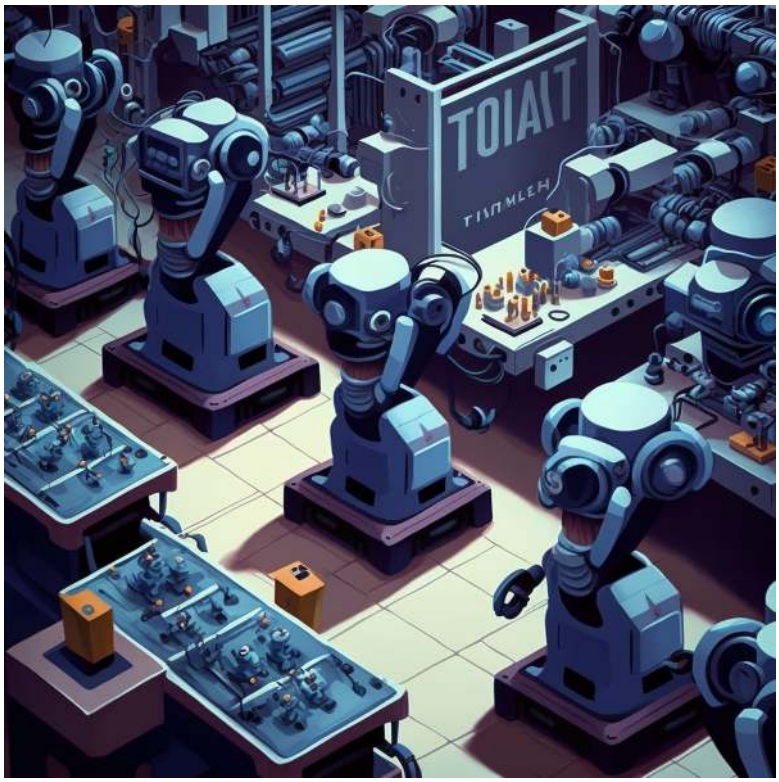
58. Clark Stanley Co. produces liniment, which requires two costly ingredients, capsaicin and turpentine. From x pounds of capsaicin and t gallons of turpentine Clark Stanley can produce $q(x,t) = 6x^{\frac{1}{3}}t^{\frac{1}{2}}$ crates of liniment in one year.

(a) Current input prices are $p_x = 100$ (\$/pound) and $p_t = 400$ (\$/gallon). What is the cost-minimizing input combination for producing 120 crates of liniment?

(b) Continued from part 58a. Clark Stanley has made a long-term contract with a medicine show to supply 120 crates of liniment every year at a price of \$100/crate. Turpentine price can vary across years. Now what is Clark Stanley's yearly turpentine demand $t^d(p_t)$ holding constant other prices?

(c) Continued from part 58a. The competitive market price of liniment is \$100 per crate. What is the profit-maximizing choice of inputs and output?

Glass bottle of liniment by Clark Stanley Co, 19th century product (Midjourney). Label for Clark Stanley Co, 19th century medicine company (Midjourney).



Tyrell Corporation robot assembly line. Fully automated robots produce robots from tungsten silicon and Gigawatts of energy (Midjourney).

59. Tyrell Corporation produces robots in just one fully automated production line, where the most recently produced robot does the work of producing one more robot before itself being shipped to customers. The other inputs are raw materials (such as silicon and tungsten) and energy. Each new robot requires a particular bundle of raw materials, but the speed of production can be increased by using more energy per robot. The production line has a maximum capacity of 4000 robots per month, beyond which overheating would destroy the facility. From x tons of raw materials and y GWh of energy Tyrell Corp can produce $f(x,y) = 10x\sqrt{y}$ robots per month. Current market prices are $p_x = 300$ (k€/ton) and $p_y = 100$ (k€/GWh).

    (a) This month Tyrell Corp. will produce 1000 robots. What is the share of energy out of the total cost of production?

    (b) In the near term Tyrell Corp. is committed to producing 1000 robots per month. Energy prices vary over time but other prices stay constant. What is Tyrell's monthly energy demand $y^d(p_y)$?

(c) Continued from part 59a. A civil war in Berserkistan reduces the supply of tungsten, causing the price of Tyrell's raw material bundle to increase to 400 (k€/ton). How does this affect the cost share of energy?

60. The technology for producing gubbins is well established: the necessary inputs are high-skill labor h and low-skill labor l. Anyone who has set up a gubbin factory can expect to produce $f(l,h) = 2l^{\frac{1}{2}}h^{\frac{1}{3}}$ gubbins per week. The current wage rates are $w_h = 6$ and $w_l = 1$ k\$/week for high and low-skill labor respectively. A gubbin factory also has a fixed cost of 1000 k\$ per week. Any one producer is too small for its output to matter for the market price of gubbins p (k\$).

(a) What is the cost-minimizing input mix for producing 100 gubbins?

(b) What is the supply function $q^S(p)$ of an individual producer?

(c) In the long run, producers can enter and exit the market. The demand for gubbins is sufficient to support a thriving industry with numerous producers. What is the market price of gubbins in long run equilibrium?

Lilliputian laborers at work. A richly detailed painting by Hieronymus Bosch (Midjourney).

61. The only input of production in the insular nation of Lilliput is labor, of which there is a total endowment of 1 million worker-years. It can be used to produce either health care or other goods. Lilliputians only value health care inasmuch it affects their life expectancy x. If h million worker-years are used in the production of health care, then

life expectancy is $X(h) = 20 + 100\sqrt{h}$. Labor can also be used to produce other goods y, with a linear technology where every h million worker-years contributes into one unit of y (measured in units of GDP per capita). Lilliputians share a common utility function $U(x, y) = (x - 20)^{\frac{1}{2}} y^{\frac{1}{2}}$.

(a) Describe the production possibilities available for Lilliput. You can show a graph or a formula.

(b) What is the GDP per capita and life expectancy at Lilliputians' social optimum?

5 Decision analysis

62. A random tourist is about to spend two days sightseeing Helsinki. There are several options: a daily ticket with unlimited travel for €8, a two-day ticket for €12, or individual tickets at €3 that are valid for one trip.

    This tourist will for sure take exactly two trips on her first day. However, she is uncertain about how many trips she will take during her second day. Her experience on the first day will determine her marginal benefit (MB) for trips on the second day. Depending on whether the experience is {great, ok, poor} the MB of her first trip on the second day is {5, 4, 3} euros. The MB of additional trips is always one euro less than the previous trip. Before the first day she views each of these experiences as equally likely.

    (a) Describe the decision facing this tourist using a decision tree.

    (b) Describe the optimal ticket-buying plan for this tourist.

    (c) Suppose this tourist could eliminate the uncertainty about her second day ticket demand by spending some effort in reading more about Helsinki. How much effort, as measured in euros, should she at most be willing to spend to achieve this?

Random tourists on a sightseeing tour, Helsinki tram (Midjourney).

63. Old Macdonald has a farm and can plant either cumin or fava beans. Fava beans would generate a profit of 180 k€ regardless of weather. Cumin would generate a profit of 300 k€ in case of favorable weather, but only 100 k€ if hit by night frost. The probability of night frost hitting is 0.5.

    (a) What is the highest expected value OldMacdonald can achieve?

    (b) Continued from part 63a. How sensitive is the optimal decision to the probability of night frost p?

    (c) Suppose it is possible to find out whether the risk of night frost is high ($p = 0.8$) or low ($p = 0.2$) this season by delaying the sowing. A delayed sowing reduces profits by 10% regardless of crop. Depict Old Macdonald's decision tree. What is the optimal course of action?

    (d) Continued from part 63c. If Old Macdonald were risk averse, how could this affect his optimal decision?

Old Macdonald fava bean farm, gothic painting (Midjourney).

64. Sirius Cybernetics Corporation is considering the launch of "Plastic Pal", a new model of human-friendly robots. However, before a model can be certified as human-friendly, it has to pass the Voight-Kampff test. This requires building a prototype, which in turn requires

first developing the blueprints for the model. The cost of developing blueprints for the new model is \$20 million. The cost of building a high quality prototype is \$35 million. The probability of passing the Voight-Kampff test is 60% if the prototype is prepared with high quality. A cheaper way to produce a prototype at a cost of \$15 million results in only 20% chance of passing the test. The test can be retaken, but the result for the same prototype quality is always the same. The high-quality prototype is always more successful: if it does not pass the test, then neither would the low-quality prototype. The demand for a certified human-friendly robot is $Q^d(p) = 1000(100 - 2p)$, where p is in \$thousands. Any number of mass-produced non-human-friendly robots can be sold, but only in the competitive world market for military robots, at a price of \$20,000. Mass production would require building a dedicated production line at a cost of \$200 million. A robot production line has a capacity to produce 400,000 robots at a marginal cost of \$25,000.

(a) Illustrate the situation facing Sirius Cybernetics with a decision tree. What is the optimal plan of action for Sirius and its expected value?

(b) Marvin Consulting Inc has the ability to predict the outcome of the Voight-Kampff test just based on the blueprints. How much, at most, should Sirius be willing to pay for its services? Consider only a deal where it gives its prediction for both prototypes.

(c) How sensitive is the optimal plan of action in 64a to the number of potential buyers N, which is associated with the demand $Q^d(p|N) = N(100 - 2p)$? I.e., at what level of N does the decision change, and how?

(d) How, if at all, does the answer to 64a change if the test results with high-quality and low-quality prototypes are independent? I.e., without the assumption that a failure with high-quality implies failure with low-quality?

65. Production company Puupäät Oy has come up with a notion for a new television serial. Developing the notion into a concept would cost €1.5m. New concepts have an equal chance of being either good or not. If they are not good, there is an equal chance of being either brilliant or bad.

The cost of turning a concept into an idea is €2m of which the production company could pay for only half, so an outside investor would be required to move forward. An outside investor must be promised 50% of the revenue in case the idea is sold. Investors cannot observe quality, only revenue.

Team workers brainstorming new ideas, surrealist painting (Midjourney).

Finally, an idea can be sold to a major network at a price that depends on the state of the economy and the quality of the idea. In a normal economy a high-quality idea can be sold for €24m, a low-quality idea for €9m, and an ordinary no-quality idea for nothing. In a booming economy the prices of ideas double; this has a one-in-three chance. Brilliant concepts would always turn to high quality ideas, whereas the merely good concepts never turn out high quality and are twice as likely to turn out low rather than ordinary quality. Bad concepts have 25% chance of turning into high-quality ideas; else they have no quality.

(a) Draw the decision tree facing the production company. What is the optimal decision and its expected value?

(b) The share (%) of revenue that must be promised to outside investors could be different. At what point (if any) would it change the optimal decision of whether to develop the notion into a concept?

(c) What would be the value of a perfect forecast about the state of the economy to the production company at the initial stage in this decision?

66. The pharmaceutical company PanaceaGenix is facing a decision of whether to begin a R&D project for a drug that would treat congenital anomie. The R&D phase would cost €1 billion. (All monetary values are in present value.) There is a 50% chance that the R&D phase succeeds; failure means that no candidate drug is produced. A drug can be sold only if it passes three successive clinical trial phases. In order to start a phase trial all previous phases have to have been successful. Phase 1 is pass/fail, it costs €0.25 billion and fails with 20% probability. Each phase 2 and 3 costs €0.5 billion and has three equally likely outcomes, dependent on how well the drug performs relative to placebo: failure, weak success, and strong success.

    Producing the drug would cost €20 per person-year dose; fixed costs are negligible. The demand for treatment is $Q^d(p|s) = 400s - 5p$, where the quantity is in millions of person-years, p is the price of a yearly dose, and s is the number of trial phases with strong success (demand is negligible if $s = 0$). This demand is aggregated over the 20-year lifetime of a patent. After patent expiry numerous generic manufacturers would sell the drug at marginal cost.

    (a) Draw a decision tree that describes the situation faced by PanaceaGenix. The decision tree should be on its own page, separate from any calculations.

    (b) Describe the plan of action that maximizes the expected present value for PanaceaGenix.

    (c) How sensitive is the decision to begin the R&D project to the probability of Phase 1 failure? I.e., at what probability, if any, would the optimal decision flip?

67. Continued from question 66. In response to popular outcry, the government institutes a windfall tax on pharmaceutical companies. In particular, pharmaceutical companies will have to pay a 50 % tax on their excess profits, which the government defines as the difference between price and marginal cost. Assume that the government can observe the marginal cost. What is the impact of the windfall tax on PanaceaGenix's optimal course of action in the following cases?

    (a) The drug has just passed Phase 2 and 3 trials with strong success. The windfall tax is instituted as a complete surprise.

    (b) The windfall tax is instituted before the decision to invest into R&D.

68. Cheezy is a modestly profitable webstore selling premium quality cheese to connoisseurs around the continent. The management is considering a change to the landing page, which is the page shown to new visitors. (Repeat customers see a different page). Currently, the default imagery depicts Gouda, a conventional and safe choice, but very similar to the branding of numerous online competitors. The new idea is to swap the Gouda for Casu Marzu, which would give Cheezy a more distinctive brand and, it is hoped, convert a larger share of first-time customers into repeat customers. However, the edgier Casu Marzu imagery is also expected to turn away some consumers, and it is not clear which effect is stronger.

    Cheezy could implement a randomized experiment starting one week from today. During the experiment, half of new visitors would see version A (Gouda), while others would land on version B (Casu Marzu). In a normal week, Cheeyz's wage costs are €80k. The experiment would require overtime from the staff, which would incur a cost of €40k.

    Currently, Cheezy gets 10k new visitors a week. The conversion rate (CR) is 10% (i.e., 10% make a purchase), and the average profit is €20 per purchaser. The repeat purchase rate (RPR) is 50%, that is, half of those who made a purchase become repeat customers. The additional present value of profits yielded by a repeat customer[10] is €100, while others never return to the site. Cheezy's analytics team estimates that there is a 20% chance that version B will double the CR, and a 80% chance of the CR being cut in half. If the impact on CR is positive there is then a 50% chance that the RPR will increase to 75%.

    A one-week experiment has sufficient statistical power to reveal the impact of the experiment on both CR and RPR. While CR is observed at the end of the week, any impact on RPR can only be detected one week after the experiment has ended. Outside the experiment, Cheezy has only one landing page, which it can change at most once a week. Five weeks from today, the landing page will be changed to something completely different due to the impending holiday season. Any information gathered before the holiday season will be worthless thereafter in the fast-moving business of online premium cheese.

    (a) (20p) Draw a decision tree that describes Cheezy's decision problem. The decision tree should be on its own page, separate from any calculations.

    (b) (20p) How valuable is the option to begin this experiment?

[10] This is also known as the customer lifetime value (CLV).

(c) (10p) How sensitive is the decision to begin the experiment to the probability x at which option B improves also RPR in the case where it improves CR?

Programmers developing novel software at a start-up, looming product launch, threat of failure concentrates the minds (Midjourney).

69. Many firms are in the business of creating novel software products. A successful product launch yields a value of 100 m€, before taking into account payments to programmers. Launching a product requires completing $n = 5$ different development tasks. A start-up will be successful only if every task is successfully completed. Due to time pressure these tasks have to be executed simultaneously; if any one of them fails then the launch fails. Firms must hire a separate programmer for each task. The ability of a programmer is measured by the known individual probability p_i of succeeding in a development task.

    Any number of average programmers, those with $p_i = 0.9$, can be hired by paying them their opportunity cost 100 k€. Programmers who are known to be above-average, i.e., those with $p_i > 0.9$, are scarce and their pay levels are determined in competition between firms.

(a) What is the expected gross value (i.e., value before wages) of a launch in a firm that relies on average programmers? What about a firm that hires programmers of above-average ability $p_i = 0.91$?

(b) Continued from 69a. The competitive pay level of programmers is such that the expected value of a launch, net of wages, is the same regardless of which of these two types of programmers a firm hires. Also, those with the same ability earn the same pay. What is the pay level of these above-average programmers?

(c) Continued from 69b. Over time new software projects become twice as big and twice as complex: now a successful launch yields 200 m€ and requires $n = 10$ development tasks. How does this change affect the earnings of above-average programmers?

70. O.y. Bonk A.b. is considering an expansion of its lucrative anchovy oil business to the Baltic countries. Entry to a new country requires setting up a distribution network and an initial marketing campaign. This cost of entry is €600,000 per country, although, if a distribution network is wound down, then all but the marketing cost of €20,000 can be recovered. Bonk faces an opportunity cost of capital at 10 %. The yearly demand for Bonk's anchovy oil in each of the Baltic countries is known to be $P(q) = 125 - 0.05q$, where quantity is measured in barrels and price in €/barrel. Bonk can produce enough anchovy oil at a marginal cost of 10 €/barrel to more than satisfy any realistic level of demand. The marginal cost of distribution is known to be 12 €/barrel in Estonia. However, in Latvia, due to a more uncertain regulatory environment, the marginal cost of distribution is deemed to be equally likely either 2 or 22 €/barrel. The actual marginal cost would be found after a year of sales. Anchovy oil is banned in Lithuania, so entry there is out of question.[11]

(a) What is the expected present value of entry in each country?

(b) How much should Bonk be willing to pay to find out the actual distribution costs in Latvia prior to committing to a market campaign there?

71. Unikorni Inc is planning to introduce a new widget. It has so far paid $15m in development costs. The widget would produce a yearly net revenue (i.e., revenue net of variable costs) of N ($m). Widget production incurs a fixed cost of $10m per year. The relevant discount rate is 5%. According to Unikorni's best estimate N is equally likely to be either 4, 8, or 12 $m. Launching the product will reveal N after one year of sales. Alternatively, Unikorni

[11] Additional irrelevant information is available here: http://bonkcentre.fi/en/businessjananchovy.

could wait for more external information to arrive before commencing production; it would then find out N, but this would delay the possibility to launch the product by two years (i.e, until year $t = 2$).

What is the optimal decision and its value in the following scenario?

(a) It is known that starting 4 years from now (year $t = 4$) the widget is obsolete and so could no longer yield revenue.

(b) The widget would never become obsolete.

(c) In the future there is a 5% probability every year that the widget becomes obsolete. Production decision for the year is made after possible obsolescence becomes known.

Three scientists solemnly sign the founding charter of their company. Two females, one male. Plastic diorama. (Midjourney).

72. A partnership of three scientists is pondering the launch of a startup that would develop a new device. If the device is developed they can sell the startup for €2.5 million. The cost for the R&D phase would be 200,000 € per scientist for labor and materials. The development of the device is uncertain, so the R&D effort may end up going to waste. There are two scenarios. Either developing the device is "hard" in which case the probability of R&D success is $p = 0.2$, or it is "easy" in which case $p = 0.8$. Both scenarios are considered equally likely.

Is the launch profitable in expectation? And how much should the partners be willing to spend, at most, on finding out whether developing the device is "easy" or "hard" before commencing with the R&D?

(a) p applies to whole R&D process.

(b) p applies to each scientist separately, each scientist has their own simultaneous task, and for the R&D to succeed at least one of them has to succeed.

(c) p applies to each scientist separately, each scientist has their own simultaneous task, and for the R&D to succeed every task has to succeed.

(d) p applies to whole R&D process, but, instead of there being two equally likely scenarios, any $p \in [0, 1]$ is deemed equally likely.

6 Strategy

73. Acme and Becme compete for customers in a market where every customer yields €1k of net revenue. There are 100k potential customers in the market, each of whom will choose to buy from the firm whose advertisement they see first. The share of customers who observe Acme first is equal to Acme's share of total advertising spending in the market. In particular, if both spend the same amount then both get half of the customers.

    (a) Suppose the only possible amounts of advertising spending are $x \in \{0, 25, 50\}$ € million. Write down the payoff matrix of this game, and find the Nash equilibrium.

    (b) Firms can spend any amount $x > 0$ on advertising. What happens in Nash equilibrium?

    (c) In addition to affecting market shares, advertising also determines the total size of the market. The customers are first attracted to the market by advertisements of both firms, so that the number of customers is $n(x) = 32\sqrt{x}$ thousands, where $x = x_A + x_B$ is in € millions (e.g., with a €4m of total spend the customer base is 64k). However, the customer base divides their eventual purchases between the firms in proportion to their advertising expenditures. Advertising spending can only take on the values assumed in part 73a.[12] What happens in Nash equilibrium?

74. Acme and Bonk are the only producers of garum. They have to decide how much garum to produce for the next holiday season. The key ingredient in garum is cured anchovy. Both firms can obtain anchovy by sending vessels on fishing expeditions, which each yield enough anchovy for 8 tons of garum. For Acme the resulting variable cost is 20 €k/ton. For Bonk, thanks to its superior curing technology, the variable cost is only 10 €k/ton. Both firms face a fixed cost of garum production of 200 €k/season. They make production choices before finding out each others' choices. Consumers find their output indistinguishable. The demand for garum is $P^D(Q) = 100 - 2Q$, where Q is in tons and P in €/kg. A firm can send out 0, 1 or 2 vessels.

    (a) Write down the payoff matrix (in €k).

[12] Alternatively, you can solve the version where any amount $x > 0$ can be spent on advertising. This requires numerical methods.

(b) What happens in Nash equilibrium?

(c) Unbeknownst to Acme, Bonk has acquired industrial espionage capabilities that allow it to find out Acme's choice before making its own decision. Would it be in Bonk's interest to disclose this capability to Acme before Acme commits to its choice?

75. Alice and Bernard are neighbors and the small patch of forest between their cottages is home to delicious chanterelles. Both value the consumption of freshly picked chanterelles at 16 €/liter, and both have an effort cost of picking of x^2 €/h where x is the picking speed at l/h. A total of 24 liters of chanterelles grows in the patch. Both start picking at the same time, choose their picking speed simultaneously and continue at the same speed until the patch is empty of chanterelles. (A player that does no picking gets nothing).

(a) Write down the payoff matrix of a game where both choose their picking speed from the discrete set $\{0, 2, 4, 6\}$ l/h. What is the Nash equilibrium of this game? What would be socially efficient?

(b) Suppose both Alice and Bernard put some weight on each other's welfare. In particular, their payoffs are now $u(v_i, v_j) = (1 - \beta)v_i + \beta v_j$, where v_i is one's own and v_j the neighbor's "selfish" payoff, and $\beta = 0.25$ is a parameter capturing the strength of social preferences. Now what is the Nash equilibrium?

(c) Suppose Alice is a faster picker, incurring only the effort cost $0.5x^2$ €/h. Assuming that speeds are selected from the same set as in part 75a, now what is the Nash equilibrium? What would be socially efficient?

(d) Suppose Bernard publicly commits to a picking speed before Alice makes her choice. Assuming that speeds are selected from the same set as in part 75a, now what is the equilibrium of the game?

76. Alpha Inc and Beta Corp are the only companies capable of mining unobtainium, which is found in the asteroid belt. The cost of building an asteroid mining ship is 4 trillion euros. A single-use mining ship can bring down to earth an asteroid with q tons of unobtainium at a further cost of 2 trillion euros per ton. The demand for unobtainium is $Q(p) = 60 - 12p$ tons, where p is in trillions of euros.

(a) The companies decide on the size of their mining operation before finding out each others' choices. What are their profits in equilibrium?

Corporate Logo of Alpha Inc, unobtainum mining company (Midjourney). Corporate Logo of Beta Corp, green and orange mining company (Midjourney).

(b) Beta Corp has the capability of building a more advanced mining ship, but it would add 2 trillion to the cost. The benefit would be that it would bring down the marginal cost to 1.5 trillion €/ton. Beta would have the option of hiding this investment (and the fact that it even had the option) from Alpha. Should Beta make this investment?

(c) As in part 76a, but Alpha gets to launch its ship first. Beta sees the capacity of Alpha's ship before deciding on its own.

(d) Suppose Beta made the investment in part 76b. Now one company launches its ship first. The other company sees its capacity before deciding on its own capacity. What are equilibrium profits and how do they depend on which company launches first?

77. Retailers can buy trinkets from wholesalers at 1 €k/m^3. The fixed cost of running a trinket retail shop is 10 €k/month. The demand for trinkets in a town with a population of Nk trinket enthusiasts is $Q^d(p|N) = (12 - p)N$ m^3/month, where p is in €k/m^3.

(a) Smalltown has two trinket retailers and 2k trinket enthusiasts. In Cournot-Nash equilibrium, what are firm profits and the retail price of trinkets?

(b) Continued from 77a. In the long run, retailers can enter or exit the trinket business. How many retailers are there in long-run equilibrium? What are firm profits and the retail price of trinkets? Compare industry profits with part 77a.

(c) There are many towns, with populations of trinket enthusiasts varying between 2k and 10k. In what range does the equilibrium number of trinket retailers and the price of trinkets vary across these towns?

78. Ann's Cafe and Bob's Bakery compete in the local confectionery market with their signature cakes, which are viewed as imperfect substitutes by most local consumers. The demand for Ann's cakes is $Q_a^d(p_a, p_b) = 240 - 15p_a + 10p_b$, where prices are in euros and quantity in cakes per day; the demand for Bob's cakes is symmetric (i.e, just flip a and b). Both have a capacity to produce up to 200 cakes per day at a marginal cost of 20 €/cake. The cost of this production capacity is 500 €/day for both.

(a) Suppose prices are chosen simultaneously. What are prices and profits in equilibrium?

(b) Suppose Ann is able to publicly commit to a price before Bob makes his choice. What are prices and profits in equilibrium?

(c) Suppose Bob is able to publicly commit to a price before Ann makes her choice. What are prices and profits in equilibrium?

(d) Which prices would maximize the combined profits of Ann and Bob?

Town square view of Ann's Cafe and Bob's Bakery (DALL-E).

79. Ann's Cafe and Bob's Bakery are neighbors. Many local connoisseurs buy their coffee from Ann's and their pulla from Bob's, before enjoying them together on the common seating area on the town square. The demand for Ann's coffee is $Q_A^D(p_A, p_B) = 2400 - 200p_A - 100p_B$, where prices are in euros and quantity in units per day. The demand for Bob's pulla is symmetric (i.e., just flip A and B). Both have a marginal cost of €2 per unit, and daily fixed costs of €500.

    (a) Suppose prices are chosen simultaneously. What are prices and profits in equilibrium?

    (b) Suppose Ann is able to publicly commit to a price before Bob makes his choice. What are prices and profits in equilibrium?

    (c) Suppose Bob is able to publicly commit to a price before Ann makes her choice. What are prices and profits in equilibrium?

    (d) Suppose Ann and Bob were to merge their businesses. Which prices would maximize the profits of Ann & Bob? Why is the impact of the merger on consumers so different from the old problem set question 78?

80. The unscrupulous Mr Montana is planning to engage in illegal tax avoidance. He could hide $1 million of cash savings in the vault of one of his three mansions: A, B or C. The local tax authority is aware of these vaults and of Mr Montana's total lack of scruples, but has the permission from a judge to raid only one mansion (if no hidden cash is found then no further permissions are granted). The cost of conducting an inspection is $200k, and any hidden cash that is found is transferred to the authority.

    (a) Before entering the simultaneous choice situation with the tax authorities Mr Montana has to decide whether to hide his cash or to just to pay the tax on it. If he pays his taxes the authorities will see this in time and there will be no point in raiding his compound. What happens if the proportional tax rate is $\tau = 0.4$?

    (b) Continued from part 80a. Consider the impact of any proportional tax rate $\tau \in (0, 1)$. What is the expected value to Mr Montana as a function of the tax rate τ?

    (c) There is an additional penalty, whereby a mansion that is found to be used for hiding cash is forfeited to the tax authority. The tax authority knows that Mr Montana values the mansions differently. His favorite mansion, A, has value $3 million, while B and C are worth 2 and 1 $million respectively. Each mansion would sell at the same $1 million in the market, which is the value to the tax authority. What is the payoff matrix of this game? Find the Nash equilibrium.

Unscrupulous Mr Montana engaging in tax fraud, New Yorker cartoon (Midjourney).

81. There are a number of e-scooter sharing companies in Lintukoto. Now is the winter season when the fleets of e-scooters have been removed from the streets for yearly maintenance. This costs a company 15 €m if it is to continue offering its services in Lintukoto next year. By now it has become clear that if n companies operate in Lintukoto then each of them earns a profit of $\pi(n) = 24/n$ €m (not including the yearly maintenance cost).

    (a) There are 2 companies (A and B) contemplating whether to stay in or stay out for the next season. Express the game in payoff matrix form.

    (b) Continued from part 81a. Describe the Nash equilibrium or equilibria.

    (c) As in part 81a, but there are 3 companies (A, B and C).

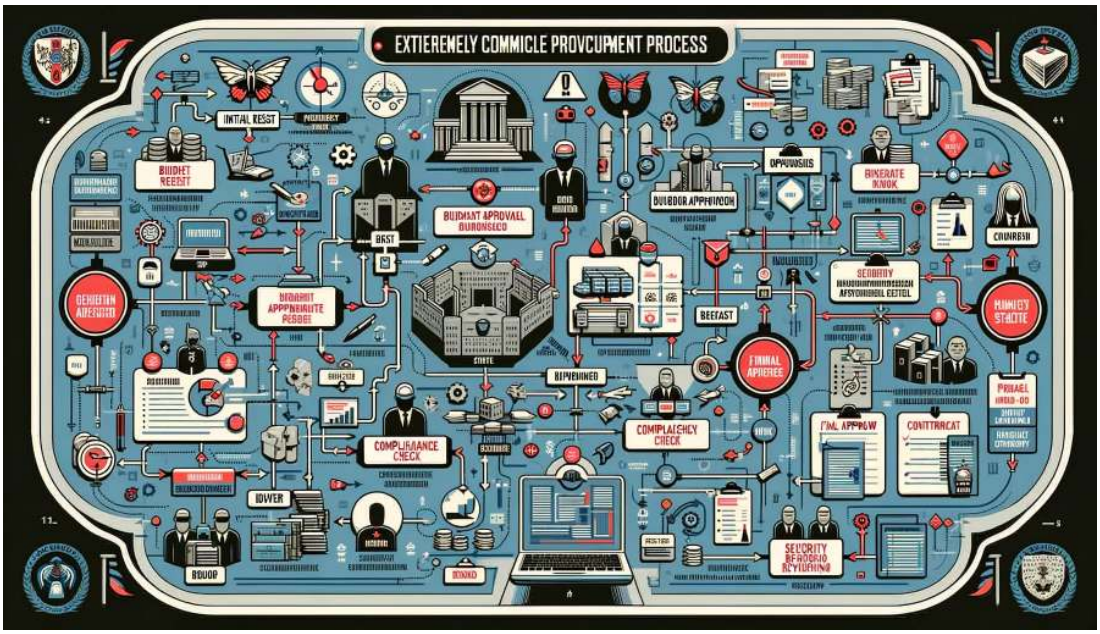
    (d) Continued from part 81c. In the symmetric Nash equilibrium, what are expected profits and what is the probability that all stay in?

82. Two electric scooter companies are making a decision between staying in and exiting from Northland. If there are n firms in the market then each makes a yearly revenue of $R(n) = 100/n$ €million at a fixed cost of 60 €million. Firms commit to paying the fixed cost once a year before finding out each others' decisions. Not paying the fixed cost even for one year results in irreversible exit. All firms use the discount rate $r = 0.05$.

(a) Suppose the market is about to enter its final year, after which scooters are known to become obsolete. What is the Nash equilibrium?

(b) What would the highest feasible present value of profits for the industry?

(c) What is the symmetric equilibrium of the repeated game, and the associated expected present value of profits for the industry?



Infographic in landscape format: The extremely complicated procurement process (DALL-E).

83. The government of Ruritania solicits offers to build a new bridge. It uses a very onerous and arcane method of procurement: to have a chance of getting a construction contract a company has to prepare detailed plans in advance and pass many regulatory reviews, just for its offer to be admissible. This costs a total of 6 billion Ducats to a company. If multiple companies present admissible offers then the government awards the contract randomly to one of them; if none do, then the bridge will not be built. A company that gets the contract will eventually be awarded 10 billion Ducats by the government, and incurs the actual construction cost of 2 billion Ducats. Companies make their choices independently without knowing what other companies have decided to do.

(a) Consider the case with only two construction companies. Write down the payoff matrix. What are the expected profits of a company? How likely is it that the bridge gets built?

(b) Continued from 83a. The Ruritanian government wants to be absolutely certain that the bridge gets built. If the only thing it can do is to increase the amount of Ducats awarded to the winner, then how much is needed?

(c) Solve the case for $N = 3$ companies. Describe the symmetric Nash equilibrium. What are the expected profits of a company? How does the additional firm affect the probability that the bridge gets built?

84. Firms Xenon and Ytterbium are the only producers of stuffium. Both have a fixed cost of 15 m€/year and a constant marginal cost of 17 €/kg. The yearly demand for stuffium is $Q^d(p) = 400 - 2p$ million kg, where price is in €/kg. Firms make their yearly production decisions unaware of each others' decisions.

a) Describe the equilibrium: price, profits, and consumer surplus.

Owners of Xenon and Ytterbium are considering a merger. What would be the impact of the merger on total profits, consumer surplus, and total surplus if the merged firm, X&Y,...

b) can save on total fixed cost due to organizational economies of scale. The fixed cost would be 16 m€/year.

c) suffers from some organizational diseconomies of scale. The new fixed cost would 40 m€/year.

d) has no impact on total fixed costs, but can achieve a lower marginal cost, 14 €/kg, due to complementarities in cost-reducing R&D.

85. A group of five villagers consumes food produced on two types of lots: a private lot, where a villager keeps the whole crop for themselves, and a common lot, where the crop is divided equally between all villagers. The individual decision is how to divide one's time between farming one's private lot and farming the common lot. The output of a private lot is $y_i(t) = \sqrt{t}$ units of food, where t is the amount of time spent farming it. The output of the common lot is $y(T) = 5\sqrt{T/5}$ units of food, where $T = \sum_i t_i$ is the total amount of time spent on farming the common lot. Each villager is endowed with 10 units of farm work time. Villagers maximize their personal consumption of food.

(a) Write down the payoff function $V_i(t_i, T_{-i})$ of an individual villager i, where t_i is the time spent by i on the common lot and T_{-i} is the total time spent by all other villagers on the common lot.

(b) Which allocation of work time between activities would maximize total output of food in the village?

(c) Write down the best response function $\text{BR}_i(T_{-i})$ of an individual villager. How much food is produced in Nash equilibrium?

Brand advertising, OneGulp blueberry soda (Midjourney). Brand advertising, TwoSips blueberry soda (Midjourney).

86. There are two blueberry soda brands, OneGulp and TwoSips, and they compete on prices. Many soft drink aficionados have their own favorite among the two and are willing to pay a premium to purchase their favored brand instead of the other. The demand faced by OneGulp is $Q_1(p_1, p_2) = 20 - 2p_1 + p_2$, where p_2 is the price charged by TwoSips; the demand faced by TwoSips is symmetric. Quantities are in thousands of liters, prices in €/l. Both firms face a marginal cost of 5 €/l.

(a) Suppose both firms commit to a price before finding out the competitor's price. What are the prices and profits in equilibrium?

(b) Suppose OneGulp is known to commit to a price at a certain point in time. If TwoSips can choose whether to commit to its price before or after OneGulp, which should it choose? Price commitments are observed by competitors.

(c) Continued from 86a. Due to a drought near TwoSips' production facility it has to haul in blueberries from further away, increasing its marginal cost to 6.5 €/l. This

problem becomes public before the prices are chosen. How does the problem faced by TwoSips affect the price and profits of OneGulp?

87. Consider the blueberry soda duopoly of OneGulp and TwoSips in Problem 86 but repeated without end in sight. Due to the structure of contracts with wholesalers the firms have to commit to prices once per year. It suffices to consider just three prices: the Nash equilibrium price p^N, the collusion price p^* that maximizes joint profits, and the best response to the latter aka "the cheater's price" p^C.

    (a) Which prices would maximize joint profits?

    (b) Write down the payoff matrix for the one-period game aka "the stage game", with the payoffs rounded to the nearest whole number in €k.

    (c) Use the payoff matrix from 87b. Both firms have a discount rate of 5%. Can they sustain collusion?

    (d) Use the payoff matrix from 87b. Both firms have the same discount rate $r > 0$. What is required of r for collusion to be sustainable?

88. Consider the earlier one-time marketing game between Acme and Becme in 73a. Now the same situation is repeated once per year, with no end in sight. Both firms have a yearly discount rate $r = 0.1$.

    (a) Show how it is possible to have an equilibrium where firms moderate their advertising spending to the level that maximizes total industry profits.

    (b) Suppose firms commit to advertising spending as part of their long-term brand strategies, which last for $t \in \mathbb{N}$ years at a time. How long can this decision period be for spending moderation to be sustained in equilibrium?

89. Consider the one-time interaction between Alice and Bernard in Problem 75 with a choice of picking speed from 0, 2, and 4 l/h. The same situation is repeated every autumn. Both neighbors have a yearly discount rate $r = 0.1$.

    (a) Both Alice and Bernard expect to remain neighbors for five more years. What is the subgame perfect Nash equilibrium?

    (b) Both Alice and Bernard expect to remain neighbors forever. How (if at all) can they achieve a higher payoff, every period, in an equilibrium of this repeated game than what they received in the Nash equilibrium of the one-time interaction?

(c) Suppose that, every autumn, the chanterelles only appear with a 50% probability. Picking only commences if there are chanterelles. How does this change the answers to parts 89a and 89b?

(d) Suppose Bernard is less patient than Alice, with $r > 0.1$. How impatient does Bernard have to be for the answer to part 89b to change?

Two mushroom hunters, Alice and Bernard, facing off in the forest (Midjourney).

90. The City of Hipsturbia has 100 000 residents, located evenly along its 10 km long main avenue. Hipsturbians have simple preferences over gelato: they value gelato at 10 €/liter, but are willing to consume only one liter per week. They also experience a shopping cost of 5 €/km for the distance from home to shop.

Gelato artisans have a simple cost structure. The weekly cost of running a shop, including rent and labor, is € 25k. The variable cost of materials is 4 € per liter of output.

(a) The first gelato shop locates near the center and faces no competition. What price maximizes its profits? How far away does it attract customers from?

(b) Eventually gelato shops spring up all along the main avenue. The price of gelato becomes standardized at 7 €/liter; any shop deviating from this price would be either derided for price-gouging or under suspicion of cutting corners; either way, a

devastating social media campaign would force a deviator to exit. How many gelato shops are there in equilibrium if shops are evenly spaced from each other?

(c) Continued from part 90b. One gelato shop obtains the capability of changing its price. It has competitors in both directions which are still forced to charge the standard price. (Locations are fixed.) Would it benefit from cutting or raising its price?

91. Arcturus and Bellatrix are the only licensed pharmacies in Linetown. The license gives the right to operate one pharmacy. All buildings in Linetown are located along its 1-km-long main avenue, and its 100k inhabitants live evenly spaced along it. Every inhabitant buys one unit of medicines per year. They minimize total cost, which is a combination of the cost of medicine and a travel cost $10x^2$ €, where x is distance between home and point of purchase in km. Pharmacies obtain the medicine from a wholesaler at a unit price €5 and have no other variable costs.

(a) Pharmacies do not compete on price, which is regulated at €10 per unit. Where do they locate the pharmacies in equilibrium?

(b) Continued from part 91a. It becomes possible for new entrants to obtain a license. Now it must be taken into account that pharmacies face a yearly fixed cost €120k. Describe a symmetric equilibrium, where no more pharmacies can profitably enter, existing pharmacies cannot benefit from exiting, and the distance between each neighboring pair of pharmacies is the same.

(c) Arcturus is located 1/4 km from the western end and Bellatrix 1/4 km from the eastern end of Linetown. They compete on prices. What are the equilibrium prices of medicine? Compare consumer welfare with part 91a. If Arcturus could change its location, to which direction (if any) would it move?

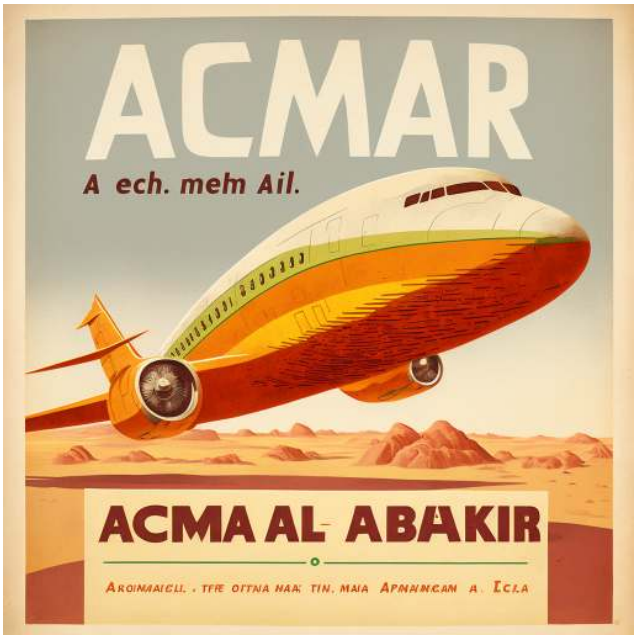
92. AcmeAir and BonkWings are the only airlines that could operate on the Lintukoto-Pohjola route. The demand on this route is $Q^D(p) = 200 - 10p$, where quantity is the number of trips per day and p is the ticket price in monetary units (MUs). There are two plane types. The cost of serving the route (MU/day) is $C_1 = 300$ for a small plane and $C_2 = 500$ for a large plane; the capacity constraints are $N_1 = 60$ and $N_2 = 100$ daily trips respectively. Marginal costs are negligible. One airline can have at most one plane on this route. AcmeAir has a blue logo and BonkWings has a yellow logo.

Companies face the decision of with which plane type, if any, to operate this route. If two airlines operate the route, then both will operate at full capacity and the price is

determined accordingly. If one airline operates the route, it can set the price as it wishes. The decisions to operate a route are made annually. Consider the decisions for one year, and measure the objective as average daily profits.

(a) (10p) As a foreign company, AcmeAir is not allowed to operate on domestic routes, leaving BonkWings a monopolist. What is the quantity of trips and the ticket price?

(b) (20p) A new Open Skies agreement makes it possible for AcmeAir to operate on this route. However, as the incumbent, BonkWings has the ability to make its irrevocable operation decision before AcmeAir. What is the quantity of trips and the ticket price in equilibrium?



Advertisement for BonkWings Inc, Cheap flights to Mos Eisley (Midjourney). Advertisement for AcmeAir, Cheap flights to Spin Boldak (Midjourney).

93. AcmeAir and BonkWings are the only airlines competing on the Mos Eisley – Spin Boldak route. Every day there are 800 potential travelers, each with a preferred departure time. These preferred times are distributed uniformly between 08:00 and 18:00. Customers value a trip at their preferred departure time at \$400, but every hour of deviation reduces the value by \$40. It is not possible for two flights to depart at the same time. Departure slots are available at quarter-hour intervals, the first slot at 08:00 and the last at 18:00.

    A plane can carry up to 500 passengers at a fixed cost of a \$40k per flight, while the marginal cost of adding passengers is negligible. The airfare is fixed by regulators at \$200 (discounts are not allowed either). Everything works the same in both flight directions, so it suffices to consider just one direction.

(a) Both airlines have the capacity to serve this route for at most one flight per day. AcmeAir gets to choose its departure time first having been the first to enter this market in the past. Describe the flight departure times and profits in equilibrium.

(b) In the long run the companies can adjust their number of flights per day. Describe the flight departure times and profits in a long-run equilibrium where no two consecutive departures are operated by the same company. In equilibrium neither company can profit from adding or discontinuing one flight. (No need to consider rescheduling existing flights.)

(c) Describe a flight schedule that would maximize industry profits.

(d) Describe a flight schedule that would maximize total surplus.

Aerial view of Donut City, built on a circular island (DALL-E).

94. Donut City is built on a circular island with a circumference of 10 km. It has a population of 1 million, with homes evenly distributed along the circle-shaped main boulevard. Every resident buys one unit of medicine per year. They minimize total cost, which, in addition to the expense on medicine, includes a travel cost $10x$, where x is the distance (km along the boulevard) from home to pharmacy. Pharmacies buy medicine from wholesalers at a price of €5 per unit, and have a fixed cost of €1 million per year.

(a) The price of medicine is regulated at €10 per unit. The government has granted licenses to four pharmacies. They are all located equally far from their competitors on both sides. What are total industry profits? What is the total cost faced by average consumer?

(b) The price of medicine is regulated at €10 per unit. There is free entry of profit-maximizing pharmacies. How many pharmacies are there in symmetric equilibrium? What are total industry profits? What is the total cost faced by average consumer?

(c) Continued from 94b. The regulator lowers the price of medicine to €8 per unit. What is the impact on consumer welfare in the long run when firms can enter and exit?

95. Abholos and Bokrug are the only ice cream vendors on Shell Beach, which is a straight 1000 meter long stretch of shoreline. It is uniformly populated by 1000 sunbathers, all of whom have unit demand for ice cream at reservation value €5 and who also experience a shopping cost of €0.50 per 100 meters of distance from their beach location to the vendor. Marginal cost of ice cream is €1. Prices are posted in increments of €0.01 and possible vendor locations are 1 meter apart.

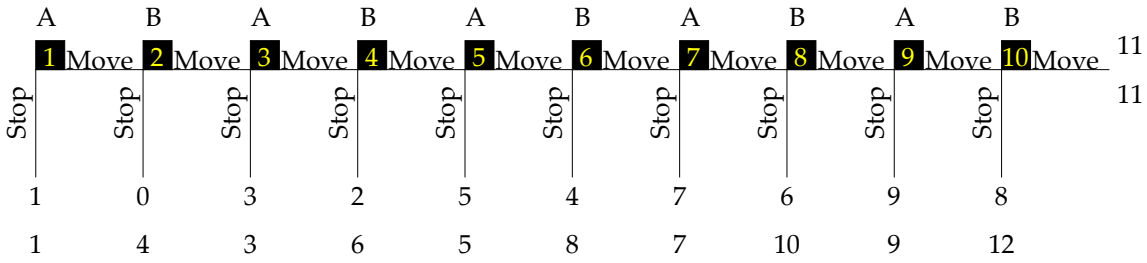
(a) Abholos is located 300 meters from the western end of Shell Beach while Bokrug is located at 400 meters from the eastern end. If both charge €2.00 for ice cream, what are their profits? Use a graph to describe the total consumer surplus.

(b) Continued from 95a. Suppose Abholos can relocate, while knowing that Bokrug cannot. Where should it move?

(c) Continued from 95a. Suppose Bokrug can change its price, while knowing that Abholos cannot. How should it price its ice cream?[13]

96. Consider the centipede game below. Which strategy would maximize your expected payoff in the following case? In part 96a consider both cases where you are in the role of player A or player B; otherwise consider yourself as player A.

[13]Hint: pay attention to two special prices. At one price level B will start losing "captive" customers at the eastern end of the beach. At another threshold level it will suddenly lose all remaining customers to A.



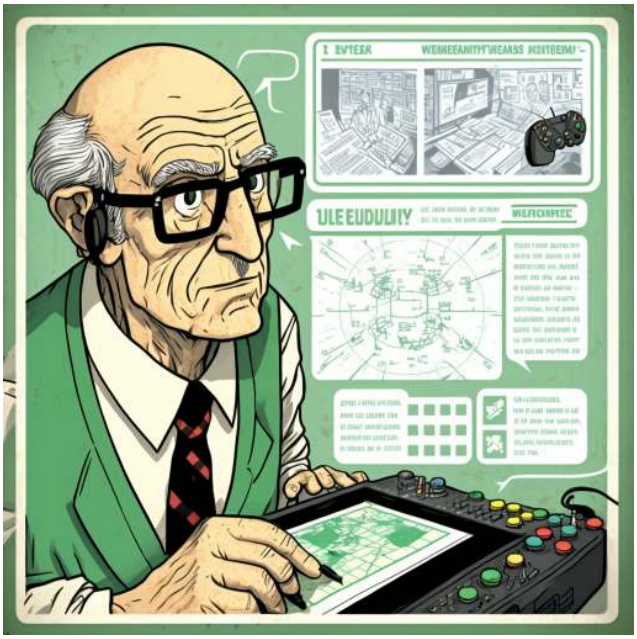
A variant of the centipede game.

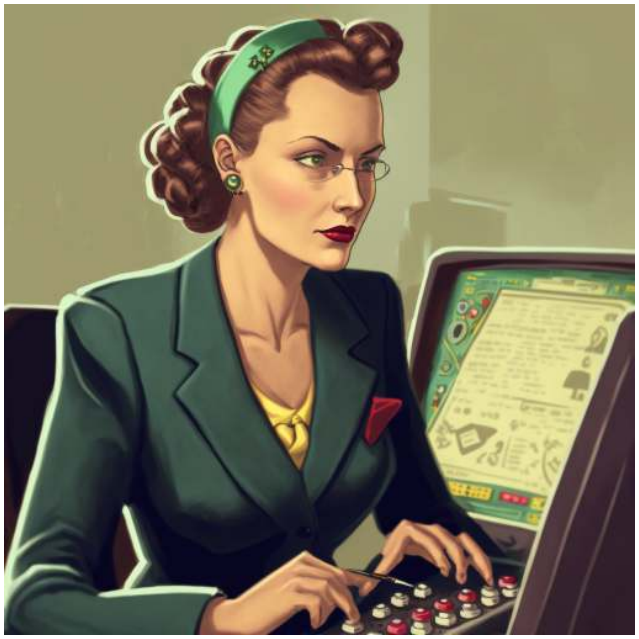
(a) The other player is a hardheaded game theory enthusiast, and assumes that so are you. They will maximize their expected payoff every step of the way while assuming that you do likewise.

(b) The other player is clueless and behaves randomly. At every opportunity they have a 50-50 chance of stopping or moving.

(c) The other player is clueless and behaves randomly. They choose every available pure strategy (every possible point of first stopping including "never") in the whole game with the same probability.

(d) As in part 96a, but the hardheaded game theory enthusiast has social preferences, and they incorrectly assume likewise about you. The game tree shows monetary payoffs, but their utility is in fact a weighted sum of your and their monetary payoff where their own payoff gets 75% weight and yours 25%. (In case of indifference between choices the enthusiast randomizes with equal probability.)





Hard-headed old-school game theorist, preparing for strategic interaction (Midjourney). Hard-headed old-school female game theorist, preparing for strategic interaction (Midjourney).

7 Pricing

97. You have been put in charge of pricing at a corporate division that leases robots to customers all over the world. The operations of your division are fully automated, with a fixed cost of €6m per day. The division controls a fleet of 300k robots. Every leased out robot incurs a cost of €50 per day, while idle robots incur no costs.

    Potential customers consist of 1000 security companies. Most of them are estimated to be ordinary users; their demand for renting robots is $q_1(p) = 200 - p$, where quantity is in robot-days and price in €. However, it is estimated that on any given day a quarter of the security companies are actings as fronts for warlords, causing their demand curve to shift to $q_2(p) = 200 - 0.5p$. It is not possible to figure out which security companies are acting on behalf of a warlord at any given time.

    (a) Your boss is an old-fashioned CEO who wants you to price the rentals like it's always been done before: your decision is to set the daily rent that maximizes profits. What rent would you set, and what would be your profits?

    (b) You attempt to convince the CEO that there is a better way to price the robot rentals, even though you cannot tell the customer types apart. Describe your strategy. Explain briefly to the old-fashioned CEO why it works better than basic pricing.

    (c) Continued from part 97b. It turns out that robots that end up serving warlords suffer additional wear and tear that increases your costs by €x per day over the original estimate. Conduct a sensitivity analysis on $x > 0$. Would you change the pricing strategy if x is sufficiently high?

98. Warre's Diner offers a buffet lunch. There are two types of potential customers in its neighborhood, hipsters and normies, with respective individual monthly demands $Q_h^d(p) = 24 - p$ and $Q_n^d(p) = 12 - 0.5p$, where q is the number of buffet visits and p the price in euros. The neighborhood has 100 hipsters and 200 normies. The cost of serving one buffet visit is €4. Warre's has a fixed cost of 10 k€/month for organizing the buffet.[14]

    (a) What is the profit-maximizing simple price for the buffet?

[14] Some additional irrelevant information on Warre's buffet: https://www.kaleva.fi/sarjakuvat/fingerpori/2227111.

(b) Warre's Club Card gives members access to the buffet at a price that is not available to non-members. Design the profit-maximizing pricing scheme.

(c) Continued from 98b. Compared to simple pricing, what is the impact of Warre's Club Card on the consumer surplus of hipsters and normies respectively?

(d) Turns out that hipsters consume so much guacamole that the cost of serving them is €6/visit. How does this affect the optimal pricing scheme in part 98b?

Warre's Diner, indoor view of diner counter, buffet lunch, 1960s Wartburg style advert (Midjourney). Logo for Warre's Diner, 1960s Wartburg style (Midjourney).

99. The vehicle dealer Oy Kärry Ab is the only seller of new Studebaker automobiles in Lintukoto. It has a contract with the manufacturer that allows it to purchase Studebakers at €115k per vehicle. The yearly demand for new Studebakers in Lintukoto is $Q^D(p) = 800 - 4p$, where the price is in €k. For each vehicle that Kärry deals it incurs a cost of €5k. Kärry also has a fixed cost of €3m/year for each automobile brand that it keeps in its product line. Kärry is currently selling Studebakers at €160k.

(a) Kärry executives know that the manufacturer knows Kärry's cost structure and the local demand for its cars. They also know that the Lintukoto market provides a tiny fraction of the manufacturer's worldwide sales, so it is reasonable to assume that its marginal cost is essentially constant. Show how they can infer from the available information that the manufacturer's marginal cost is €35k.

(b) Which retail price in Lintukoto would maximize the combined profits of the dealer and the manufacturer?

(c) Kärry executives decide to make an offer to the manufacturer that would increase the profits for both. This involves paying a yearly license fee in exchange for a right to buy vehicles at a reduced wholesale price. Design an offer that, if accepted, would maximize the combined profits and divide the increase in profits equally between the dealer and the manufacturer.

100. Lämpölintu Oy provides district heating for local residents in the Lintukoto metropolitan region. The yearly demand for heat by a single-family home resident is $P_1^D(q) = 120 - 1.5q$ and $P_2^D(q) = 70 - q$ by an apartment building resident; quantities are in Convenient Thermal Units (CTUs) and values in €/CTU. There are 10,000 potential customers of each type. The marginal cost of heat is constant 20 €/CTU, up to a capacity constraint of 1.4 million CTU/year. The yearly cost of maintaining a district heating connection is 500 €/year per customer. The fixed cost of the power station is 10 million €/year.

Describe the profit-maximizing pricing scheme, and the resulting profits and aggregate consumer surplus in the following cases.

(a) Uniform pricing of CTUs.

(b) A connection fee and a price per CTU.

(c) A connection fee that may be different by the building type of customers, same price per CTU for all.

(d) A connection fee and price per CTU that may both differ by building type.

101. Acme Ltd produces breakfast gruel. The cost of producing one portion of gruel is €2. It can be mixed with water to produce thin gruel at half the marginal cost. Potential customers are either health-conscious (H) or low-income (L), both equally numerous. Either would buy at most one portion of gruel at a time. Customer valuations are ...

	Gruel	
€	Thin	Thick
L	1.50	2.90
H	1.20	2.30

(a) Describe the profit-maximizing pricing scheme.

(b) The share of health-conscious customers begins to increase. How much higher can it get before the optimal pricing scheme from part 101a changes?

Television commercial for Thin Gruel, health-conscious breakfast (Midjourney). Television commercial for Thick Gruel, affordable hearty breakfast (Midjourney).

102. A company can deliver packages as either Regular or Overnight. The cost of delivery is 5 €/parcel, regardless of speed. Potential customers have a million packages a week to send. For 50% of the parcels the sender is in a hurry. Customer valuations are...

	Speed	
€	Regular	Overnight
In hurry	15	40
Not in hurry	12	20

(a) What is the profit-maximizing pricing scheme?

(b) The Consumer Protection Agency finds out that Overnight service is as cheap to produce as Regular service, and bans Regular speed delivery as an affront to consumers. What is the impact of the ban consumer surplus?

(c) It turns out that Regular delivery is in fact more expensive to provide, as it requires more storage space while parcels wait in their destination cities before final delivery to recipients. How expensive, in €/parcel, can Regular service be before the optimal pricing scheme from part 102a changes?

(d) The share of parcels that are in a hurry begins to decline. How much lower can it get before the optimal pricing scheme from part 102a changes?

(e) Half of those not in hurry become so laid back that they value delivery at any speed at only €10. The company has the capability to introduce an extra slow SuperSaver delivery, which other types value equally with Regular delivery and which would also cost 5 €/parcel to provide. What is the profit-maximizing pricing scheme?

Parcel delivery service, regular or overnight, classic advertising (Midjourney).

103. A popular cellphone manufacturer releases a new model. There are two equally common types of potential buyers, ordinary folks and techies. The manufacturer is planning to release a special version with added features in addition to the the basic version. Techies value the extra features by more. The variable cost of a phone is €100 regardless of the features. The valuations are

€	Basic	Special
Ordinary	300	360
Techies	450	720

(a) Describe the profit-maximizing pricing strategy.

(b) The manufacturer could add a large logo onto any phone model, at an additional cost of €5 per phone. Ordinary customers don't care about the logo one way or another, but it would annoy the techies and reduce their valuation by €50. Describe the profit-maximizing pricing strategy.

(c) Continued from 103b. How does the optimal pricing strategy depend on the impact that the logo has on techies' valuation? Denote this impact as the parameter X (assumed -50 above).

Logo for Jacque's gourmet chocolatier, 1920s design (Midjourney). Logo for Jacque's gourmet chocolatier, sleek 21st century design (Midjourney).

104. Jacque's Chocolatier produces boxes of confectionery. He knows that there are two types of customers with a different demand for pieces of its gourmet chocolate: chocoholics with $Q_c(p) = 50 - 10p$ and ordinary types with $Q_o(p) = 30 - 6p$, where quantities are in pieces of chocolate and values in €. There are 100 ordinary types and twice as many chocoholics. The marginal cost of production is 0.50 €/piece; all other costs are sunk. Jacque can easily verify whether someone is a true chocoholic by asking a few expert questions as part of the normal store counter chit-chat, and so could potentially offer them a discount. However, anyone can pretend to be an ordinary type by not answering.

(a) What are the profit-maximizing sizes and prices for Jacque's boxes of chocolate?

(b) A worldwide supply crunch hits cocoa prices particularly hard, with Jacque's marginal cost almost tripling to 1.40 €/piece. He responds by the optimal amount of "shrinkflation". Now what are the box sizes? Compared to part 104a, which customer type ends up with higher inflation (% change in average price)?

(c) Continued from part 104b. In the short run Jacque cannot change the box sizes, but can change the prices or discontinue a box size. Now what is Jacque's optimal response to the increase in cocoa prices? How does consumer welfare differ (in aggregate, and by individual type) from the long run case where box size can also be adjusted?

105. A detergent manufacturer has both professional and industrial customers. There are 1000 industrial customers each with demand $Q_1(p) = 840 - 40p$, and 3000 professionals each with demand $Q_2(p) = 300 - 15p$, where quantities are in kg and values in €. Production incurs a fixed cost of €2m, while marginal cost is 10 €/kg.

(a) Design the profit-maximizing pricing scheme and package sizes.

(b) Continued from part 105a. Conduct a sensitivity analysis on the number of industrial customers. At what point is the manufacturer better off selling only one package size?

106. Acme food truck is known for two specialties, grilled pineapples and grouse stew. It has three types of potential customers, 100 of each type. Their valuations for Acme's servings are...

€	Grouse	Pineapple
Bourgeois	15	13
Students	6	11
Workers	14	9

The marginal cost of portions is €5 and €3 for grouse and pineapple respectively. The goods are neither substitutes nor complements to the consumers. What is the profit-maximizing pricing scheme and resulting profits if Acme were to use...

(a) basic pricing.

(b) pure bundling.

Bourgeois feasts on pineapples and grouse stew, satirical cartoon (Midjourney).

(c) mixed bundling.

107. Your firm is selling two products, X and Y. For purposes of price optimization, the estimated joint distribution of customer valuations has been divided to 30 equally common customer types. Estimated customer valuations are listed in xy_customers.xlsx.[15] Marginal cost of production is zero.

What are the profit-maximizing prices and resulting profits if...

(a) X and Y are priced separately?

(b) pure bundling is used?

(c) mixed bundling is used?

As always, make sure to explain the reasoning behind your calculations. Part 107c can only be solved numerically (in a reasonable amount of time).[16]

[15] https://aalto-econ.fi/tervio/kandimikro/xy_customers.xlsx.

[16] This problem is best solved numerically. In case you use Excel Solver: when choosing the solving method, notice that this is a non-smooth problem.

8 Externalities

108. The creatures on planet Zorg have recently discovered space flight, and are sending more and more satellites to low-Zorg orbits (LZO). Every additional satellite raises the risk of collisions between satellites, and every collision raises the risk of a chain reaction where the debris from previous collisions causes ever more collisions and more debris until all satellites on LZO have been destroyed; the resulting cloud of debris would render LZO unusable.

    Zorgian space scientists have calculated that if there are n satellites in LZO then the probability of a catastrophic chain reaction is $p(n) = 0.5 \times 10^{-6} n^2$ if $n < 1000\sqrt{2}$ and $p(n) = 1$ otherwise. The expected (private) net benefit from sending up a satellite is 10 Altarian Dollars ($Alt) if there is no chain reaction and -10 $Alt if there is.

    (a) In the absence of any planet-wide restrictions on the use of LZO, how many satellites will Zorgians send up? What is the expected value generated by the satellites?

    (b) What number of LZO satellites would maximize expected total welfare?

    (c) Zorgian economists propose to create a planet-wide tax on LZO satellites. What level of the tax would maximize expected total welfare?

Satellites orbiting planet Zorg, Kessler syndrome, Alien art (Midjourney). Alien space scientist at work, preventing Kessler syndrome on planet Zorg (Midjourney).

109. Bluefin tuna is a popular delicacy in the nations around the Great Sea. Most of the sea lies outside territorial waters, so there are no fishing restrictions. The cost of sending a tuna fishing vessel to the open sea is 20k monetary units (MUs). The yearly catch of any single fishing boat x in tons is decreasing in the number of boats n, so that $x(n) = 80 - 0.2n$. The market price of tuna is determined by its more abundant substitutes, and is 2k MU/ton.

    (a) What is the efficient number of tuna fishing boats, the associated total yearly catch, and the resulting total profits of the tuna industry?

    (b) In the absence of any restrictions on the entry of tuna fishing boats, how many will enter? Compare the profits and the tuna catch, both in total and as the average per-boat, with their efficient levels.

    (c) The Great Sea nations decide to take control of the open sea in order to prevent overfishing. They decide to auction off fishing licenses, so that a boat is only allowed to fish tuna if it buys a yearly license. In order to maximize total welfare, how many licenses should be sold? What would be their market price?

Old map of the Great Sea (Midjourney). Bluefin tuna, vintage illustration (Midjourney).

110. There are two alternative routes between Easton and Weston, and every day 10 000 drivers travel between them. Travel time on the Expressway is $T_1(n) = 30 + (n - 5000)/50$ minutes if $n \geq 5000$ drivers choose the Expressway, and 30 minutes otherwise. On the Highway travel time is $T_2(n) = 45 + (n - 500)/100$ minutes if $n \geq 500$ drivers choose the

Highway, and 45 minutes otherwise. Roads are owned by the government. Driver utility is decreasing in travel time: drivers value each saved minute on the road by €0.20.

(a) How many drivers choose each road in equilibrium? What is the average travel time?

(b) Design the welfare-maximizing road pricing scheme, in which drivers have to pay a toll on one of the roads. What are its welfare effects? How much does it decrease the average travel time between the cities?

(c) A popular outcry against road pricing causes the government to exempt low-income drivers from the optimal fee derived in part 110b. Half of drivers are classified as low-income. Now what is the average travel time? Compare total welfare, as well as travel time and welfare for both low and high income drivers separately, with what is obtained in part 110a and in part 110b.

111. Visiting national parks is one of the popular pastimes for the citizens of Lintukoto. There are 200k citizens who like to visit a national park at most once a year. They value the experience of pristine nature but dislike crowding.

For each of the following cases: i) What is the equilibrium (numbers of visitors, total welfare) in the absence of entry fees? ii) What is the welfare-maximizing entry fee and its impact on visits and welfare?

(a) There is one national park in Lintukoto. If n thousand citizens visit the park during a year, then each visitor values a visit by $V_a(n) = 200 - 2n$ euros. Visitors incur a private travel cost of €16.

(b) As in part 111a, but there is also another park, where Lintukotoans value a visit by $V_b(n) = 300 - 2n$ euros. (Entry fee may differ by park).

(c) As in part 111a, but consider the impact of a 50% reduction in the per-visit travel cost.

112. Two firms, Acme and Becme, are both researching cures for diseases. Both could research either one of two diseases, Common or Rare, but have to choose just one. The profits from discovering a cure is €10 billion for Common and €4 billion for Rare for whoever discovers it first (and thus gets the patent). Profits amount to 10% of total economic value, most of which is consumer surplus. In case both discover a cure for the same disease both have an equal chance of being first. The probability that a firm discovers a cure for a disease is $p = 0.75$, regardless of firm and disease. Discovery success is independent across firms.

Describe the allocation of research efforts, and expected profits and total surplus in the following cases.

(a) The firms engage in a patent race. Acme publicly commits to its research first before Becme makes its choice.

(b) The firms engage in a patent race, and commit to research before finding out each others' choices.

(c) The firms make a joint decision on how to allocate their research efforts, with the purpose of maximizing their combined value.

(d) As in part 112a, but before firms make their choices the government can promise a monetary prize for discoverers of each cure, with the objective of maximizing total surplus. The marginal cost of public funds is 2, that is, each euro paid out in subsidies creates one euro of deadweight loss elsewhere in the economy.

Bigger car eats smaller car, underground art (DALL-E). Bigger car eats smaller car, underground art (Midjourney).

113. All households in the Lake Woebegon region have to own a car. The question is whether to buy a Normal or a Big car. Cars differ only by price and safety. A Big car costs 20 €k more than a small one. The probability of a collision with another car during the lifetime of a car is 0.1 for all cars; the collision partner is randomly selected from the population of cars in the region. If a collision leads to serious damage then the cost of an accident is 1 €m, otherwise the cost is negligible. If a Big car collides with a Normal car then only

the Normal car will suffer serious damage. If two Normal cars collide then both have a 50-50 chance of serious damage. If two Big cars collide then both suffer serious damage.

(a) What fraction of cars in the Lake Woebegon region are Big in equilibrium?

(b) What fraction of cars in the Lake Woebegon region should be Big?

(c) Suppose that Woebegonian households place a premium on the comfort of owning a Big as opposed to a Normal car, but they vary on how large this premium is. Namely, this premium is distributed uniformly between 0 and 40 €k. Now what fraction of cars are Big in equilibrium?

114. Recent invention of the telephone has prompted a company in the town of Lintukoto to consider starting a telephone network.[17] The fixed cost of setting up a network is 1.2 million markka. The cost of connecting an individual household to the network is 200 markka. All participating households will be charged the same price for a connection, at which they can make calls at no further cost.

(a) Suppose every household values every connection to other households by 1 markka. How many households N does Lintukoto need to have for the telephone network to be able to break even?

(b) Suppose households differ by their valuation for telephone connections. The valuations for every connection to other households are uniformly distributed between 0 and 2 markka. (For any one household, all of its connections to other households have the same value.) If Lintukoto has 6 000 households, then what is the profit-maximizing price of a telephone connection?

(c) Continued from part 114b. How many households N does Lintukoto need to have for the telephone network to be able to break even?

115. The center of social life in the remote town of Podunk has for long been its Country Club. Ostensibly a venue for pastimes such as golf and tennis, in fact Podunkians value it for the opportunities to meet other Podunkians informally in a relatively fancy setting. Podunk has 1000 inhabitants who value the opportunity to get to know other Podunkians at the club. Their expected valuation for potential meetings with others at the club is distributed uniformly between \$0 (the completely asocial) and \$2 (the extreme socialites).

[17] Useless additional information about the impact of the invention: https://www.hs.fi/fingerpori/car-2000002457111.html.

Podunk Country Club, golf and tumbleweed, 1950s promotional poster (Midjourney).

(For any one person, all potential meetings with other unique members have the same expected value.) The yearly maintenance cost of the club facilities is 100 \$k. There is also a cost of 100 \$/year per member for running the club.

(a) What is the profit-maximizing price of club membership?

(b) The Podunk Country Club is managed by a board of members who set the price with the purpose of maximizing total surplus. If the club makes a profit it is distributed equally back to members; making a loss is not feasible as the club has to pay its bills. How do the price and size of membership differ from the situation under profit maximization?

(c) Continued from part 115b. The population of Podunk is in decline for reasons that are unrelated to its country club. How will this affect the price of membership? It suffices to explain the direction of change.

116. A real estate management company is planning a shopping center in Hyperborea. There are 200 retailers that could potentially locate in the center. Retailers benefit from the presence of other retailers in the center, because every shop attracts customers who often end up making purchases also at other shops in the center. Every shop attracts 100 unique customers to the center. Every unique customer will visit the shop that attracted them, and in addition has a 1% chance of visiting each of the other stores in the center. (Thus,

for example, if the center had 101 shops then every customer would visit on average 2 shops.) Retailer profits before rent are proportional to the number of visiting customers.

Rents cannot be customized by retailer profitability. Which level of rent would maximize real estate company profits? How large a shopping center would it build?

(a) Every shop visit generates in expectation €v of profit for the retailer, where retailer type v is uniformly distributed in [0,20]. All real estate company costs are fixed.

(b) As in part 116a, but there is a construction cost of €1000 per shop.[18]

117. A new neighborhood, Greenfield, has been zoned on the outskirts of the growing city of Mesopolis. It is located within a reasonable distance from the urban core, but it's not yet possible to build homes there due to a lack of infrastructure. There are plenty of households that would each value living in Greenfield by €50k if the city invested €100m in its infrastructure (new bridge, tram line, etc). (This is net of all private costs, such as home construction and opportunity costs.) Potential residents also value Greenfield's nature and open views, which are reduced by home construction. If there are $n > 1000$ homes then this "crowding" deducts $C(n)$ from every household's valuation, where $C(n) = 0.001(n - 1000)^2$ euros. There will be one home for each household.

(a) Suppose the cost of infrastructure is divided evenly between all households that locate in Greenfield. How many homes should be built at minimum for the public investment to be worthwhile? What would be the optimal number of homes?

(b) The infrastructure has been completed, and home construction has begun. Greenfield's first residents know that the cost of infrastructure will in fact be spread evenly between all households in the city. The rest of the city will have 100k households, regardless of how many move to Greenfield. Suppose the first residents begin to lobby for a neighborhood size n that would maximize their own surplus. What n would they lobby for?

(c) Continued from part 117a. Residents also value the variety of local services. More residents attract more private services such as retail and restaurants, and so the consumer surplus of every household increases by €10 for every additional local household. How does this affect the answers to parts 117a and 117b?

[18]This cost of capital is of course measured in present value terms over time periods of the same length as the numbers of customer visits above.

Greenfield zoning map (Midjourney). Zoning map for the new suburb of Brownfield (Midjourney).

118. A new suburb is going to be built in the municipality of Brownfield. There are many identical lots where housing will be built. The potential residents are households that would value a home in Brownfield by $V(x, y) = 100\sqrt{x} - y$, where x is floor area of the home and y is the size of the neighboring home in m^2. Larger houses require more floors, which increases the marginal cost of construction; for the same reason, larger houses reduce the utility of their neighbors by restricting the view of the sky. The construction cost of a home is $c(x) = 200 + 0.01x^2$. There are more potential residents than could ever fit into the new suburb. All monetary values are in €k.

    (a) The lots are sold to household buyers. If everyone can build a home of any size they want, what is the home size, and what is the market value of a lot before construction?

    (b) The Brownfield zoning board can set a maximum size $\bar{x}$ for houses in the new suburb. Which size restriction would maximize the total welfare created by the new suburb?

    (c) All the lots in the new suburb are owned by the same private developer, and it can freely decide the size of the houses it builds. Which house size would maximize its profits?

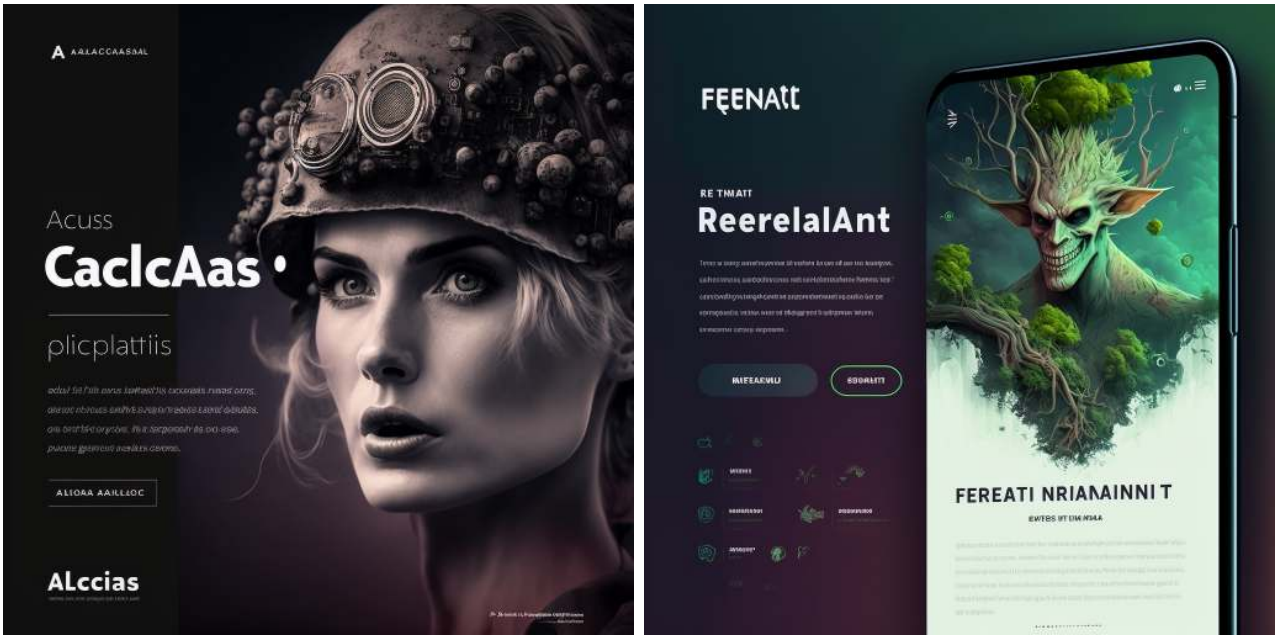
    (d) Continued from part 118b. Suppose that half of the lots have already been built, subject to the optimal size restrictions. Every second lot is still empty, so now $y = 0$. The Brownfield zoning board is deciding on size restrictions for the unbuilt lots. Which size restriction would maximize i) consumer surplus of existing residents in the new suburb, ii) total welfare, iii) revenue from selling the unbuilt lots?

119. The residents of Lintukoto conduct their non-professional networking activities on the age-old platform AllCaps. All 10 000 residents have an active AllCaps account, each with a yearly valuation $v(z, n) = z\sqrt{n}$ euros for access to the network, where n is the number of active members and z a preference parameter that depends on the quality of the user interface. AllCaps has only a fixed operating cost of €200k/year.

    (a) If all residents have a quality preference $z = 1$ then what is the profit-maximizing yearly membership fee for AllCaps?

    (b) If quality valuations are distributed uniformly, with $z \in [0, 2]$, then what is the profit-maximizing yearly membership fee for AllCaps?

    (c) Continued from part 119a. A new startup FreeRant launches a competing app with a nicer user interface valued at $z = 2$. It has initially no users, but can lure away AllCaps users by giving away swag. (Users are active in at most one platform at a time.) How many users does FreeRant need to attract in order to drive AllCaps out of business?



Front page of AllCaps, professional social media company (Midjourney). Landing page of FreeRant, noxious social media app (Midjourney).

9 Information

120. A population of technicians has lost their jobs in Ruritania due to the bankruptcy of a local tech giant, Unicorn Inc. Fortunately many other firms are now competing to hire them. Diligence is the key determinant of worker productivity in Ruritania; it is commonly known that the diligence of Unicorn technicians is varied. There are workers of three levels of diligence, all equally common. Low-diligence workers produce 200 ducats worth of economic surplus if they work for a firm, for median-diligence workers the figure is 600 ducats and for high diligence workers 1300 ducats.

    While workers know how diligent they are, it is not possible for a firm to know the diligence of any individual worker until after they have already worked in that firm for a while. Discrimination based on diligence is forbidden under Ruritanian labor law; once hired, employment at the initial wage is permanent (unless the firm goes bankrupt). Technicians can also work as independent contractors, but it is not as productive as working for an existing firm: any self-employed technician produces only 60% of the surplus they would in a firm.

    (a) Describe the equilibrium distribution of earnings (average, and earnings by type), and explain why adverse selection is or is not a problem in this market.

    (b) Continued from 120a. A few years later another Ruritanian tech giant goes bust. The situation is in other ways identical, but technological advances have improved the productivity of independent contractors. The output of all types in self-employment is now 70% of what they would produce inside a firm. (Productivity inside firms was not affected). What is the equilibrium distribution of earnings (average, and earnings by type) for this new population?

Ruritanian Ducat, rare coin (Midjourney). Ruritanian 100 Ducat bill, collectible (Midjourney).

121. An airline sells two types of tickets for a flight to the same destination. The premium ticket includes the option to cancel and get a full refund; when a buyer of a basic ticket cancels their flight they get no refund.

    The airline knows that there are three equally common types of customers in terms of their cancellation risk, and it has enough capacity to serve all of them. The cancellation risks are 15%, 5%, 1% by type. The basic ticket is priced at €100. Tickets are personal and so cannot be sold second hand. Getting to the destination is worth at least €150 to all customers. A buyer whose cancellation risk is realized experiences an annoyance cost worth €20 just from the disappointment of a wasted ticket; getting a refund removes this annoyance cost.

    What is the price of the premium ticket, and who buys what in the following case?

    (a) Suppose the airline decides to offer the option to cancel at a price that is budget-neutral: the price premiums earned exactly cover the expected refunds paid out.

    (b) The airline prices the premium ticket in order to maximize its profit.

122. There are 1000 sailboats in the used boat market. These are the boats that have passed an independent certification of being at least acceptable in terms of quality. However, there remains some uncertainty over the exact quality of these boats, which only the current owner can know. All potential buyers have the same valuations, and there are more potential buyers than there are boats. The reservation values are...

	Valuations	
€k	Seller	Buyers
Junk	15	20
Fine	20	24
Good	25	28
Perfect	32	36

    (a) All boat types equally common. How many boats are traded? Compare total welfare to a world with symmetric information.

    (b) Continued from part 122a. Suppose a boat dealer develops a superior quality evaluation technology. In particular, by paying a cost of €2k it can verify and credibly disclose the true quality of a boat. How would this impact total welfare?

(c) Fraction $x \in (0, 1)$ of boats are perfect quality, while other types are equally common. How high/low does x have to be for there to be either full market unraveling or no adverse selection at all?

Highly skilled pastry chef decorating a cake, color drawing (Midjourney). Low-skill pastry chef decorating a cake, shoddy decoration in bad taste, drawing (Midjourney).

123. Pastry chefs come in two skill levels. Highly skilled chefs make delicious cakes while ordinary chefs make plain-tasting cakes; both are equally common. Buyers value delicious cakes at €36 and plain-tasting cakes at €10. The marginal cost of production is the same €5 per cake for both, which includes the fee that pastry chefs have to pay to retail stores to get their cakes to the buyers. Buyers cannot directly observe the taste of a cake at retail stores where cakes are placed behind glass counters. However, they understand that highly-skilled chefs bake delicious cakes while ordinary chefs bake plain-tasting cakes. Furthermore, it is possible to craft complex decorations on a cake, which the buyers don't value at all because they only care about taste, but which they can see from behind the counter. Decorating cakes is painstaking work, in which the highly-skilled chefs are three times faster; for them decorating costs €10 per cake, whereas for the slower ordinary chefs the cost is €30.

(a) Show that there is an equilibrium in which cakes with complex decorations are delicious and plain-looking cakes taste plain.

(b) Suppose that buyer's valuation for delicious cakes begins to increase. At what point does the signalling equilibrium of part 123a break down? Then what happens?

124. Consider the situation facing Ruritanian technicians in problem 120a. The government decides to offer a job training program for technicians who lost their jobs in the Unicorn bankruptcy. The program is voluntary and has no impact on participants' productivity. Completing the program requires effort from the participants, and the disutility of that effort is worth 450, 150, and 50 ducats to workers of low, median, and high diligence respectively.

(a) Show that, if some employers understand the relation of worker diligence and disutility from the training program, then some workers could find it worthwhile to participate in the training program.

(b) How high can the participation rate in the training program be in equilibrium? What is then the impact of the program on the distributions of earnings and worker surplus (average, and by type)?[19]

125. There are two equally common types of high school graduates in the Woebegon Lake region that differ only by productivity. Productivity is defined by how many effective work hours θ an individual provides per an 8-hour day of time spent at work: $\theta = \{2, 7\}$ hours for {Low, High} types respectively. College in Lake Woebegon is purely focused on making its graduates proficient in Astrology and Ancient Sumerian; these skills have no impact on productivity or consumption utility.

Individuals experience a subjective effort cost that depends on the hours spent studying; lower-productivity types have to expend more effort in order to graduate so their effort cost of a college degree is higher. This effort cost is $4200/\theta$ Monetary Units (MUs) for a type with productivity θ, where 4200 is the number of effective work hours any student needs to put in to graduate. There are no other costs.

The present value of a college graduate to their employer is proportional to the effective work hours they provide, namely $v(\theta) = 400\theta$ MUs. Employers can observe neither worker type nor their effective work hours. The labor market is competitive, and equilibrium earnings leave zero expected profits to employers.

[19] If you answer this part correctly with a complete explanation, then you have automatically also answered part 124a. Thus, if you are feeling confident, you can skip part 124a.

(a) Show that there is an equilibrium where some Woebegonians get a college degree. What are the earnings by type?

(b) Due to eroding standards in Woebegonian higher education the hours of work needed to graduate begins to decrease. At what point does a college degree become useless?

(c) As in part 125a, but there is also a third equally common type and it has $\theta = 5$.

Broken gadget, cutaway drawing (Midjourney).

126. A maker of gadgets needs to signal to potential buyers that its gadgets have been built to last by offering a returns policy which promises buyers a new gadget if the gadget breaks down. Customers value a functioning gadget at €100. It costs €64 to produce one gadget. The probability of a gadget breaking down depends on how carefully the customer handles it, and this carefulness comes with an effort cost to the buyer: if a customer expends c euros worth of care then the probability of the gadget breaking down is $h(c) = \min\{1/2, 1/c\}$. The cause of a gadget's breakdown is not verifiable. You can assume that replacement gadgets are not covered by the returns policy.

(a) What would be the efficient amount of care (in €) to be expended by buyers?

(b) What is the lowest price at which a gadgetmaker could break even while offering the returns policy?

(c) An innovative gadgetmaker decides to offer a watered-down returns policy: in order to get a replacement a buyer will have to suffer a hassle cost of €x at the gadgetmaker's sluggish and inconveniently located returns department. What level of hassle x would maximize welfare? Then what would be the lowest break-even price for gadgets? The costs of running the returns department are fixed and can be ignored here.

127. Insurance companies offer accident insurance to young drivers. They prefer to drive carelessly, and value careless driving at €3000. With careless driving the probability of an accident is 0.16, otherwise only 0.04. The only damage from an accident is the destruction of the car, always worth €30 000. The same person can suffer at most one accident. Insurance companies have no other costs besides payments to insurees.

(a) How high should the deductible be for it to deter young drivers from driving carelessly? Then what would be the lowest break-even price at which insurance could be sold to young drivers? Assume that drivers are risk neutral.

(b) Continued from part 127a. What would be the lowest break-even price at which insurance could be sold to a young driver who asked for a deal without any deductible?

(c) As in part 127a, but drivers are risk averse. Their certainty equivalent for a loss that occurs at probability p is its expected value divided by $\sqrt{p}$.

128. Acme Mutual offers marine insurance to shipowners. The insurance pays out in case of total loss (e.g., if the ship sinks). The most carelessly managed ships have a 20% probability of total loss during the contract period, but shipowners can reduce this probability at a cost. There are some verifiable methods such as meeting fire standards that reduce this probability to 8%; these cost €1m and are required by Acme as a prerequisite for purchasing insurance. There are also various unverifiable methods of further risk reduction. A moderate level of unverifiable spending (€1m) would reduce the loss probability to 4%, while a high level (€3m) would reduce it to 1%.

Acme observes the true values of ships and cargo. In parts from 128a to 128c, consider two cases: a ship and cargo of "High" total value €100m and of "Low" total value €20m.

Abstract art representation of Marine Insurance (Midjourney).

(a) What would be the efficient level of spending on loss risk reduction?

(b) Suppose Acme were to offer insurance with a 35% coinsurance rate. How much would risk neutral shipowners spend on risk reduction? What would then be the cost of actuarially fair insurance?

(c) What range of coinsurance rates would incentivize a risk neutral shipowner to choose the efficient level of safety spending?

(d) Suppose Acme is offering insurance with a 35% coinsurance rate. To prevent moral hazard it decides to only insure ships of sufficiently high value. What minimum value requirement would achieve this aim?

129. An annoying disease, hydrophobia, affects the inhabitants of a distant land. It onsets in middle age, cannot be prevented, and has a very costly cure. Many private insurance companies offer coverage for this ailment. The insurance can only be purchased when young, before the potential onset. All inhabitants have a utility function $u(w) = \sqrt{w}$, where w is lifetime wealth in local monetary units (MU). The baseline wealth level is 100 MU. The onset of hydrophobia incurs a cost of 50 MU.

There are two risk types, based on a single gene: 10% of individuals in every birth cohort carry the high-risk allele, while 90% carry the low-risk allele. High-risk types have a 1/2 chance of contracting the disease, and low-risk types have a 1/90 chance.

Assume that any insurance, if offered, is full, and that insurance companies have no costs other than payouts to insurees.

(a) Initially, the inhabitants are unaware of genetic risk factors to hydrophobia, so they only know the average risk. What is the reservation price of a young inhabitant for insurance? What is the market price of insurance?

(b) Scientists discover the genetic risk factor behind hydrophobia, and a genetic test becomes available. (The test is so cheap relative to lifetime wealth that we can assume it costs nothing.) How does this innovation affect the reservation prices of those who take the test?

(c) Continued from part 129b. Insurance companies begin to condition prices on genetic test results. What are the insurance prices?

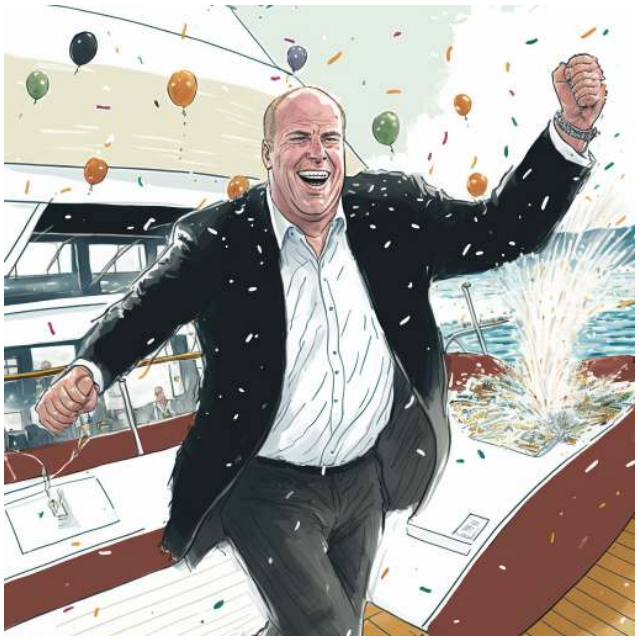
(d) Continued from part 129c. The parliament is worried about the impact of the genetic test. Three proposals are put forth; evaluate their impact on the availability and prices of insurance. Assume that each proposal, if implemented, could be perfectly enforced.

   i. Insurance companies are banned from conditioning prices on genetic test results.

   ii. Customers are banned from withholding genetic test results in case they took the test. Prices can be conditioned on test results and test status (test status = whether a person has taken a genetic test or not).

   iii. Customers are banned from withholding their test status. Prices can be conditioned on test status.

130. The luxury yacht dealer Öky-Alus Oy is recruiting a new sales rep. A sales rep that works hard has an 80% chance of selling one yacht, which would increase gross profits by €1 million ("gross" meaning profits before deducting the sales rep's compensation). If the rep takes it easy there is no chance of making a sale. In any case there is no chance of selling more than one yacht per year. The top candidate, Raymond, could also remain a sales rep at another company where he makes €100k per year while taking it easy (i.e., without working hard). The hard work needed to sell yachts would cause Raymond a disutility of effort that he values at €40k per year. The management at Öky-Alus Oy can offer a pay package with a base wage x and a bonus b for a yacht sale.

Luxury Yachts sales rep Raymond, closing the deal on the deck of the yacht, magazine illustration (Midjourney). Luxury Yachts sales rep Raymond, celebrating year-end bonus, magazine illustration (Midjourney).

(a) Which level of effort by Raymond would be economically efficient?

(b) If Raymond were risk neutral then what kind of a pay package would maximize the expected profits of Öky-Alus? If there are several alternatives that Raymond would be indifferent with then select the one with the highest base wage (this is the unique solution for an "almost" risk neutral worker).

(c) Raymond is, in fact, risk averse. He maximizes the expected value of $u(w) - c$, where $u(w) = w^{2/3}$, w is wealth and c is the cost of effort (€k). His initial wealth is $w_0 = 116$ (€k). What kind of a pay package would maximize the expected profits of Öky-Alus?[20]

131. Asfaltti Oy is formulating its bidding strategy for a highway building project. Its cost of building would be €3 billion. The highway authority uses sealed bidding in its procurement. You know that only one other firm, Raxa Group, is a relevant competitor in this auction. (There may be other bidders, but you know that their bids are certain to be higher than Raxa's). According to your best estimate, Raxa's cost of building the highway is equally likely to be anywhere between €1 billion and €4 billion. Raxa Group

[20]Hint: You can first solve for $v_1 = u(x + w_0)$ and $v_2 = u(b + x + w_0)$ as the unknowns, from which you can then solve b and x.

is managed by an old-fashioned civil engineer who insists on cost-plus pricing. You know that Raxa will always bid at cost plus 25%, no matter what the auction format. Your job is to formulate the bid for Asfaltti Oy.

(a) The contract will be awarded with a second-price auction. How much should you bid?

(b) The contract will be awarded with a first-price auction. How much should you bid?

(c) You have a shady business associate who has the capability of finding out Raxa Group's exact cost before the bidding. However, this would come at a cost. How much would this information be worth to Asfaltti Oy under each auction format?

Class photo at East Side School, typical kids (Midjourney). Class photo at West Side School, typical rich kids (Midjourney).

132. Two schools hold procurement auctions for class photos. The demand for class photos is $P_E^d(q) = 11 - q/100$ in the East Side School and $P_W^d(q) = 21 - q/100$ in the wealthier West Side School, where q is the number of photos and prices are in €. Producers vary only by their fixed costs, which arise from organizing a photo shoot in a school. All have the same marginal cost, €3, for each additional copy of the class photo once taken.

The contracts will be awarded in second-price sealed-bid auctions, one for each school. Your task is to formulate the optimal bids for a professional photographer who has a fixed cost €1000 per school. The minimum bid increment is €0.01.

(a) Bidding is for the right to be the class photo producer. Highest bidder wins, and the winner is allowed to choose the price at which they sell photos.

(b) Bidding is for the price of individual photos. Lowest bidder gets the right to be the class photo producer. In this case the school gets no revenue.

(c) Bidding is for the right to be the class photo producer. Highest bidder wins, but the winner is restricted to charge a unit price €5 for the photos.

(d) Suppose your photographer uses the optimal bidding strategy and ends up providing the second best bid. Which of the above procurement rules would yield the highest buyer side surplus (consumer surplus + school revenue) at each school?

A unique curio: an object desired by bidders with idiosyncratic tastes. (Midjourney).

133. You are bidding for a unique curio in a sealed-bid auction with one other bidder. From your point of view the competitor's valuation is equally likely to be anywhere between 0 and 300 euros, and so is yours from her point of view. Before the auction the bidders have a chance to inspect the curio, and so will find out their own valuations. You both want the curio for your own personal collection, and you both have idiosyncratic tastes. This means that figuring out your own valuation does not help you in improving your estimate of your competitor's valuation.

(a) Suppose the seller holds a first-price auction. You know that the other bidder follows a rule of thumb: always bid half your own valuation. How does your optimal bid b^* depend on your valuation v?

(b) Continued from part 133a. Suppose the other bidder believes that you will use the bidding strategy $b^*(v)$ from part 133a. Knowing this, what would be her optimal bidding strategy $B^*(V)$?

(c) Continued from part 133a. If you use your optimal bidding strategy, and your competitor follows her rule-of-thumb, then what is the seller's expected revenue?

(d) Suppose the seller holds a second-price auction. What is your optimal bidding strategy $\hat{b}(v)$? What is the seller's expected revenue if both bidders use their optimal strategies?

Two agents bargaining hard over binoculars, female and male agents, silkscreen print (Midjourney). Binoculars for agents, silkcreen print (Midjourney).

134. Two agents, Hanne and Jonne, are negotiating over whether and at what price Hanne would buy Jonne's binoculars. Time is running out. Hanne knows that reservation values of sellers like Jonne are equally likely to be anywhere between €0 and €100. Likewise Jonne knows that reservation values of buyers like Hanne are equally likely to be anywhere between €0 and €200. Both know that buyer and seller values are independent.

(a) How likely is it that it would be efficient for Hanne and Jonne to trade?

(b) Suppose Hanne makes the final take-it-or-leave-it offer, before the negotiations are over for sure. How likely is it that this leads to trade?

(c) As in part 134b, but it is Jonne who gets to make the final offer.

135. The owners of a terraced house have to decide whether to build a swimming pool in their common garden. Each of its three apartments is occupied by its owner, each owner has one vote in all decisions, and all monetary costs incurred by the house are shared equally between the owners. Building the swimming pool would cost €60k. However, the residents' valuations for the swimming pool vary. Atte values the services provided by the pool at €35k, Bette at €18k, and Citte at €14k. In the following methods for making decision, what are the total payments and surpluses by each owner?

(a) The owners use majority voting.

(b) The owners obtain the help of a psychic who can read people's minds. She sets out to decide on whether to build the pool, with the goal of maximizing total welfare. She is also instructed to devise a system of transfers between the residents with the aim of equalizing consumer surplus between owners.

(c) The owners use the Vickrey–Clarke–Groves mechanism.

(d) There is also a value to the services provided by the garden space that would be lost under the pool. These are valued at €5k by Atte, at €0k by Bette, and at €8k by Citte. Reconsider parts 135b and 135c.

Psychic reads customer mind, for true value of swimming pool, psychedelic cartoon (Midjourney).