Difference That CEOs Make: An Assignment Model Approach

Marko Terviö
Haas School of Business, UC Berkeley
marko@haas.berkeley.edu

March 2007

Introduction

<u>CEO pay</u>

Controversy over levels

Can a perfectly competitive model explain distribution of CEO pay levels?

What is the $ difference that CEOs make to welfare?

What kind of differences in CEO ability required to explain their wages?

<u>Setup</u>: One market for CEOs

I. CEOs are heterogeneous by their ability to impact surplus

II. Firms are *exogenously* heterogeneous by their surplus potential

III. No market imperfections / incentive problems

Closest related literatures

- Scale effects, Mayer (1960), Lucas (1978), Rosen (1982).
- Assignment models, Tinbergen (1956), Sattinger (1979)
- Assignment models of CEO pay Terviö (2003), Gabaix & Landier (2006)

- Others e.g.: Murphy & Zabojnik (2004), Baker & Hall (2004)

(Models of CEO pay with firm-CEO pair: numerous)

Outline

- Basic assignment model
- Adaptation to CEOs
- Inference from data
- Calibration
- Conclusion

Basic Assignment Model of Pay (adaptation of Sattinger 1979)

<u>Assumptions</u>

A1. Complementarity. Manager of type *a* and firm of type *b* produce surplus

$$Y(a,b)>0,$$

where $Y_{ab} > 0$. ➔ positive assortative matching.

A2. Continuous distributions.

Denote $a[i]$ and $b[i]$ where i is the quantile in $[0,1]$

$a'[i] > 0$ and $b'[i] > 0$ and no atoms

No uncertainty / imperfect information / frictions

Equilibrium

1. Assignments. Firm i is matched with manager i (Efficiency of CE)
2. Prices. Division of surplus to wage and profit at every firm i

$$Y(a[i],b[i]) = w[i] + \pi[i]$$

such that

no firm (or manager) can be made better off by different match:

$Y(a[i],b[i]) - w[i] \geq Y(a[k],b[i]) - w[k]$ for all i,k. "IC" constraints

$w[i] \geq w^0$ for all i Participation const.

$\pi[i] \geq \pi^0$ for all i Participation const.

Normalize $i = 0$ s.t. $Y(a[0],b[0]) = w^0 + \pi^0$

Binding IC constraints are i vs $i - \varepsilon$,

they reduce to

$$\frac{Y\left(a[i], b[i]\right) - Y(a[i-\varepsilon], b[i])}{\varepsilon} \geq \frac{w[i] - w[i-\varepsilon]}{\varepsilon}$$

Take the limit as $\varepsilon \to 0$.

$$w'[i] = Y_a(a[i], b[i])a'[i]$$

Similarly,

$$\pi'[i] = Y_b(a[i], b[i])b'[i]$$

➔ Equilibrium income distributions:

$$w[i] = w^0 + \int_0^i Y_a(a[j], b[j])a'[j]\mathrm{d}j$$

$$\pi[i] \quad = \quad \pi^0 + \int_0^i Y_b(a[j], b[j])b'[j]\mathrm{d}j$$

Division of surplus at match *i* depends on distributions of characteristics in [*0, i*].

Wage $w(a) \neq$ MP ability $Y_a(a,b)$.

A scaling lemma

Consider an across-the-board change in productivity, by factor G, while holding the distributions a and b fixed.

If $\underline{Y}(a,b) = GY(a,b)$,

$\underline{w}^0 = Gw^0$ and $\underline{\pi}^0 = G\pi^0$

➔ Factor incomes scale with G at every i:

$\underline{w}[i] = Gw[i]$ and $\underline{\pi}[i] = G\pi[i]$

Application to CEO/Firm setup

What is "exogenous component in firm size," b?

Exogenous differences = not due to differences in CEO ability

Cannot be transferred at margin between firms

- Size of firm's niche in the economy – "natural scale"
- Sunk capital, brand value

Rents that accrue to firms are capitalized into market value

Adjustable capital is not part of b – it should simply earn r

Assignment model for CEOs and firms

Equilibrium outcomes: CEO pay - flow

Market value - stock

Cannot use this data directly

- Both current and future CEOs impact market value
- Value of adjustable capital is part of observed market value

Next: Four assumptions and four parameters introduced to map the assignment model into CEO pay / market value setup.

A3: Production function. Surplus created at firm of size b, in period t

$$y_t(A_t,b) = (1+g)^t A_t b$$

where A_t is the *effective management ability* in period-t

A4: Impact of CEOs across time: Effective ability, in period t, at a firm with a history of CEO abilities $a_t, a_{t-1}, a_{t-2}, \ldots$ is

$$\begin{aligned} A_t &= A(a_t, a_{t-1}, a_{t-2}, \ldots) \\ &= \sum_{\tau=0}^{\infty} \alpha_\tau a_{t-\tau}. \end{aligned}$$

Where $\Sigma_\tau \alpha_\tau = 1$ and $\alpha_{\tau+1} = \alpha_\tau /(1 + \lambda)$, $\lambda > 0$ ==>

$$\alpha_\tau = \frac{\lambda}{(1+\lambda)^{\tau+1}}$$

A5: Strong stationarity.

- Distribution of a fixed forever,
- Firms infinitely lived, unchanging b → Distribution of b fixed forever
- Productivity at every firm grows deterministically at rate g
- Outside opportunities w^0 and π^0 grow deterministically at rate g

→ Firm b expected to keep matching with type a in the future

→ $A_t = a$ forever

→ (Present value of) Surplus to be created at firm of type b

$$Y(a,b) = \sum_{t=0}^{\infty} B^t ab = \frac{ab}{1-B}$$

where $B = \frac{1+g}{1+r}$

(A3) -- (A5) ➔ Scaling Lemma applies ➔ $w[i]$ will grow at rate g

Surplus $Y(a[i],b[i])$ is divided into

PV of CEO pay at firm i is $\frac{w[i]}{1-B}$

and

PV of profits for firm i is $v[i]$

$$\frac{w[i]}{1-B} + v[i] = \frac{a[i]b[i]}{1-B}$$

Adjustable capital

Part of market value reflects adjustable capital stock → market return *r*.

$$v^* = v + k^*$$

A6: Gross surplus has constant elasticity θ wrt adjustable capital:

$$y_t(a,b) = \max_{k_t} \left\{ \left(ab\zeta \left(1+g\right)^t \right)^{1-\theta} k_t^{\theta} - rk_t \right\}$$

(Choose units wlog: $\zeta \equiv (1-\theta)^{-1} (r/\theta)^{\theta/(1-\theta)}$ →)

$$k_t^* = \frac{\theta}{r\left(1-\theta\right)} \left(1+g\right)^t ab$$

→ multiplicative form preserved for (net) surplus: $y_t(a,b) = (1+g)^t ab$

→ value of optimally chosen adjustable capital can be removed from observed market value.

Assuming values for θ, g, r ; observed w , v^* → implied $v[i]$

After doing the math…

$$v_0 = \xi v_0^* - (1-\xi)\frac{w_0}{1-B}$$

where

$$\xi \equiv \frac{1-\theta}{1-\theta+\frac{\theta}{r}(1-B)}$$

Inference from data Assuming the model is correct…

Present value of surplus at firm b with CEO a_0, when $a_1 = a_2 = \ldots = a$

$$Y(a_0, a, b) = \sum_{t=0}^{\infty} B^t (\alpha_t a_0 + (1 - \alpha_t)\, a) b$$

$$= \frac{\lambda}{\lambda + 1 - B} (a_0 - a)\, b + \frac{ab}{1 - B}$$

Equilibrium conditions (IC)

$$w'[i] = Y_{a_0}(a_0[i], a[i], b[i]) a_0'[i]$$

Stationarity ➔ $a_0[i] = a[i]$

Equilibrium conditions ➔

$$w'[i] = \frac{\lambda}{\lambda + 1 - B} a'[i] b[i]$$

$$v'[i] = \frac{\partial}{\partial i} Y(a[i], b[i]) - \frac{w'[i]}{1 - B} = \frac{a[i]b'[i]}{1 - B} + \frac{a'[i]b[i]}{\lambda + 1 - B}$$

$$\frac{w[0]}{1 - B} + v[0] = \frac{a[0]b[0]}{1 - B}$$

Using observed *w*[*i*] and v[*i*], and assumed *λ, r, g*, we can infer

$$\frac{a[i]}{a[0]} = \exp\left\{\frac{\lambda}{\lambda + 1 - B} \int_0^i \frac{w'[j]}{w[j] + v[j](1 - B)} \mathrm{d}j\right\}$$

$$\frac{b[i]}{b[0]} = \exp\left\{\int_0^i \frac{v'[j] - w'[j]/\lambda}{w[j] + v[j](1 - B)} \mathrm{d}j\right\}.$$

Counterfactuals

Impact of replacing CEO at quantile *i* with CEO from quantile *I* for one year:

$$Y(a[I], a[i], b[i]) - Y(a[i], b[i])$$

$$= \frac{\lambda}{\lambda + 1 - B} \left(v[0] + \frac{w[0]}{1 - B} \right) \frac{(a[I] - a[i])\, b[i]}{a[0] b[0]}$$

Value of ability defined *relative to* replacement ability.

E.g., relative to baseline ability *I = 0,*

welfare impact if all top CEOs were replaced by lowest observed type.

Calibration

Data: ExecuComp 1994—2004

Largest 1000 firms in each year

CEO pay (*tdc1*) – total compensation (options with Black-Scholes)

Market value (*mtkval*)

Relation of CEO pay and Market value Lowess smoothed to make sure $w'[i] > 0$

Calibrations: 1. Counterfactuals for 2004

2. Time-series fit 1994--2004

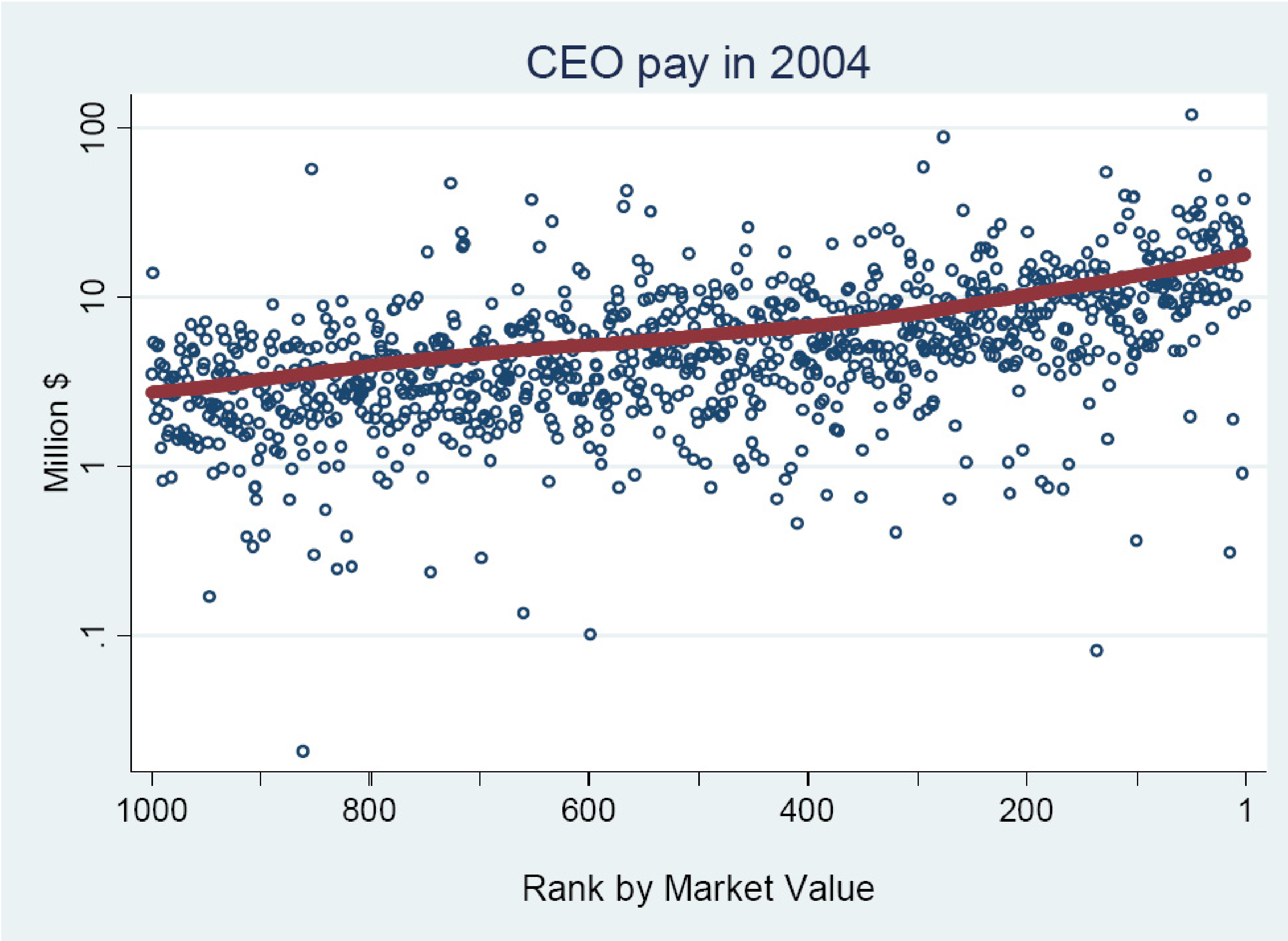


Figure 2. Relation of CEO pay and firm rank by market value in 2004. The smoothed relation (obtained with the Lowess method) appears upwards biased in the graph because the pay levels are depicted on log scale.

Parameters

$0.05 \leq r \leq 0.1$

$g \geq 0.02$

$r - 0.025 \leq g \leq r - 0.06$

→ implied "P/E" ratio $1/(1-B)$ in 17—44

θ: $0 \leq \theta \leq 0.8$

λ: ≥ 0.1 → implied half-life for influence < *7.3* years

Combinations with the most extreme results reported

Table 1. CEO ability and welfare in 2004 at top 1000 firms ($Bn)

Assumptions	(A)	(a2)	(c)	(b2)	(B)
Discount rate (r)	0.08	0.08	0.05	0.05	0.05
Growth rate (g)	0.02	0.02	0.025	0.025	0.025
Share of Adj. Capital (θ)	0	0.4	0.4	0.4	0.8
Rate of impact fading (λ)*	∞	∞	0.5	0.1	0.1

Results	(A)	(a2)	(c)	(b2)	(B)	Counterfactual rent to CEOs (A)	Counterfactual rent to CEOs (B)
Value over Baseline	24.97	24.75	23.36	23.12	21.33	0	0
Value below Max	3.16	3.17	3.25	3.27	3.40	2.85	3.36
Value over Median	7.12	7.09	6.89	6.86	6.62	1.81	2.00

Actual totals	
CEO pay	7.12
CEO rent**	4.39
Market value	12,584.6

* Results for λ = ∞ calculated from the limiting values.

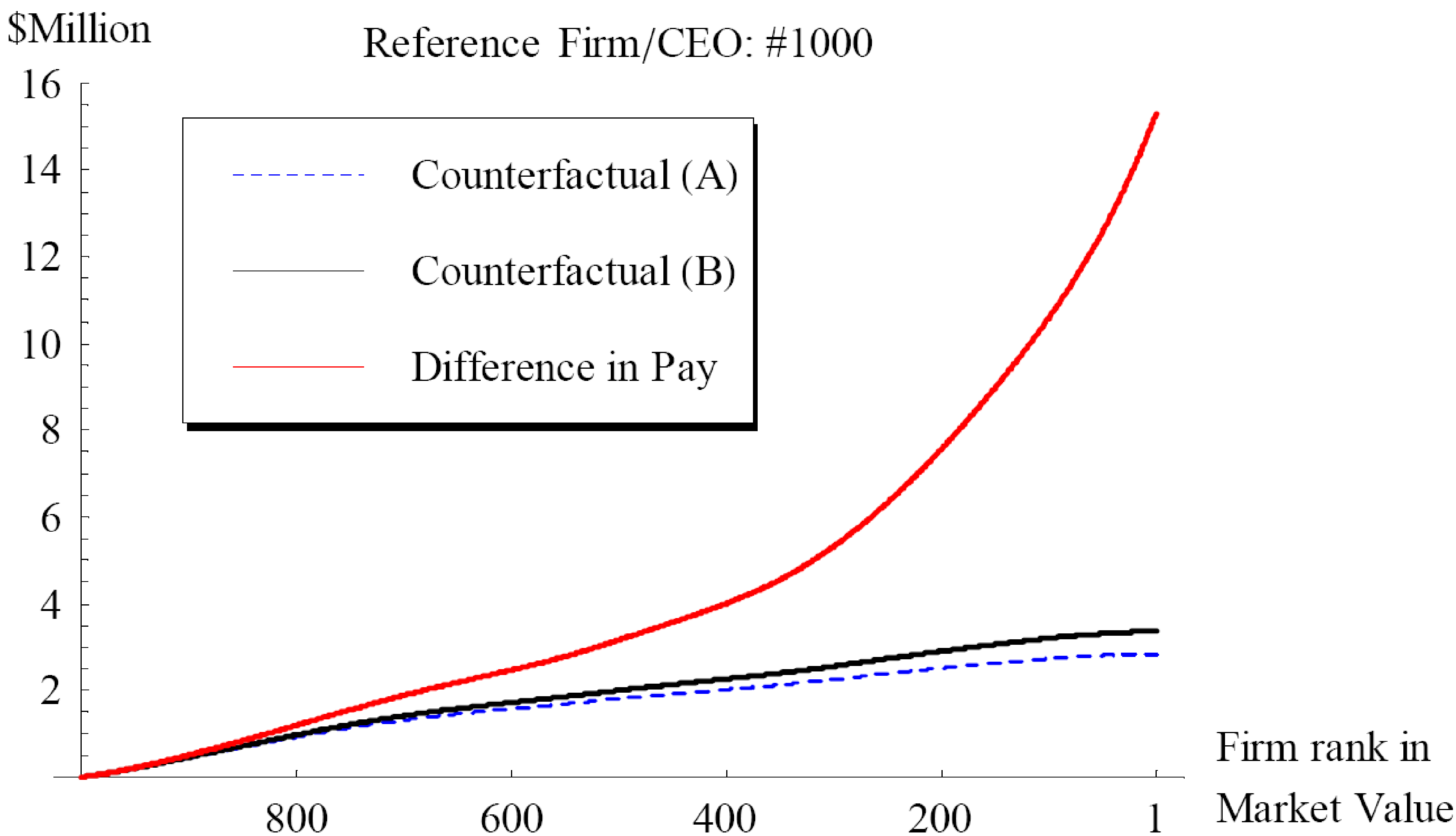
** Rent = Pay - Baseline pay ($2.7m)

Role of ability v firm size in CEO pay

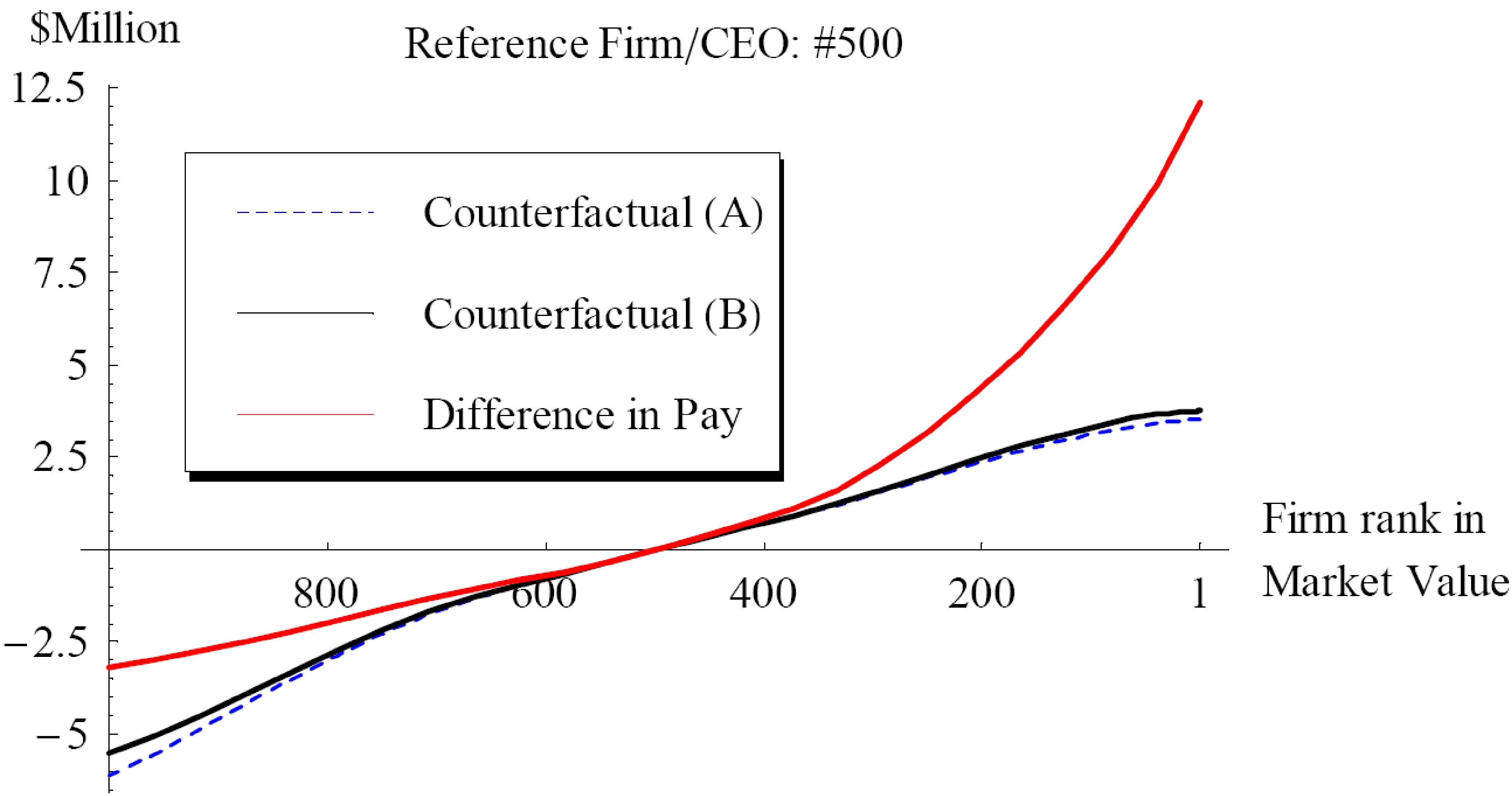
Table 2. Total rents (wage – baseline pay) to CEOs at top 1000 firms in 2004, under counterfactual firm size, $Bn

						Counterfactual Market Value	
Reference firm	(A)	(a2)	(c)	(b2)	(B)	(A)	(B)
1000th largest	1.70	1.71	1.78	1.79	1.92	1,221.9	6,673.6
500th	5.77	5.74	5.59	5.55	5.30	4,140.5	8,945.4
1st	516.4	510.3	469.3	462.5	413.3	370,663.3	108,255.

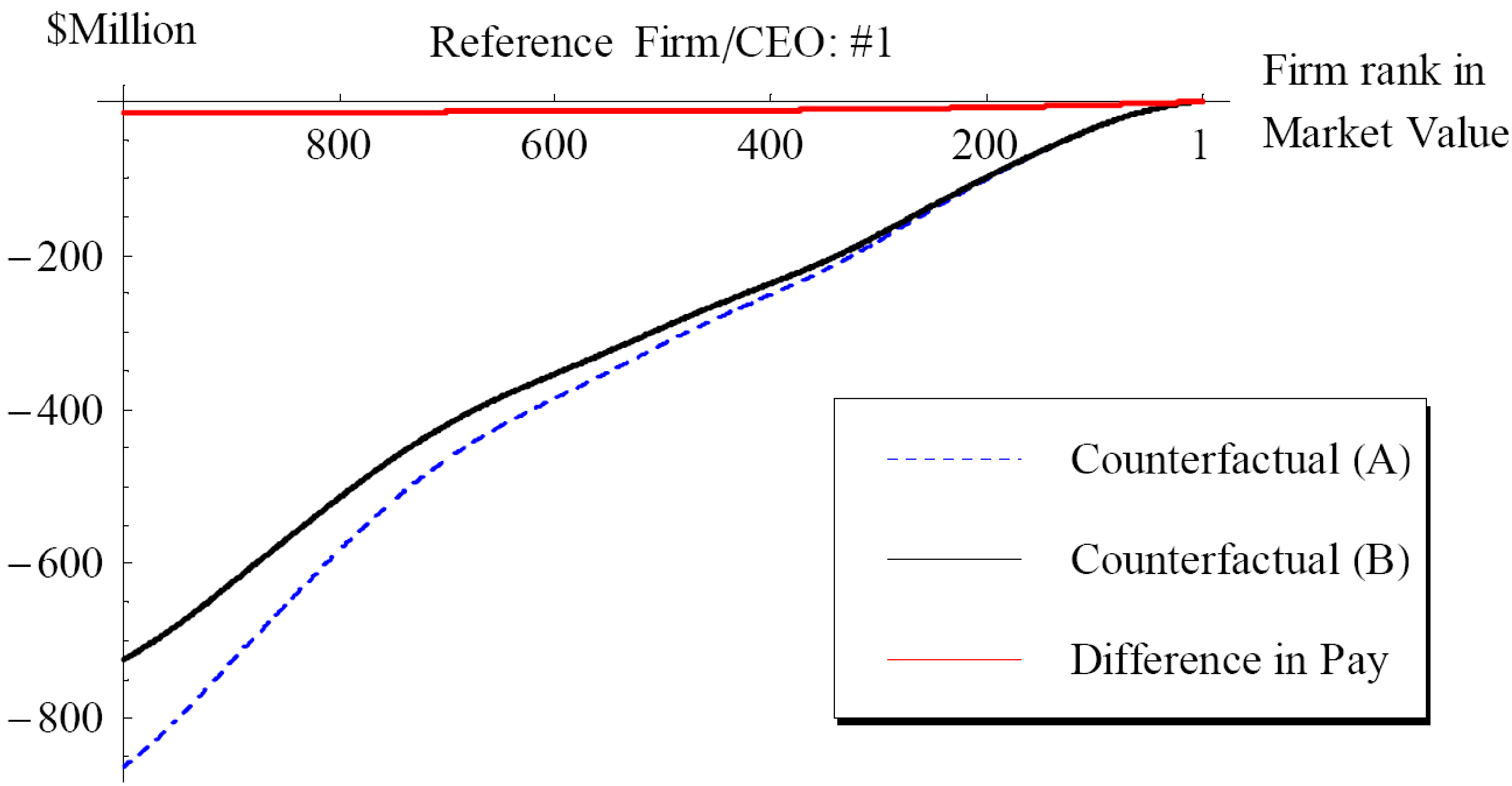
Actual totals	
CEO rent	4.39
Baseline pay × 1000	2.73
Market value	12,584.6



Impact of CEO ability at 1000th largest firm.



Impact of CEO ability at 500th largest firm.



Impact of CEO ability at largest firm.

CEO pay distributions in 1994—2004

Forcing unchanging distributions of *a* and *b*

$$Y_t(a,b) = G_t ab$$

Can this model generate the time-variation of CEO pay distribution?

1. Use average $a_t[i]$ and $b_t[i]$ inferred from cross sections

$$\hat{a}[i] = \frac{1}{T}\sum_t \frac{a_t[i]}{a_t[0]} \quad \text{and} \quad \frac{\hat{b}[i]}{\hat{b}[0]} = \frac{1}{T}\sum_t \frac{b_t[i]}{b_t[0]}$$

2. Set G_t such that total surplus in each year will fit perfectly

$$G_t \int_0^1 \hat{a}[i]\hat{b}[i]\mathrm{d}i = \int_0^1 \left(\frac{w_t[i] - w_t[0]}{1-B} + v_t[i] - v_t[0] \right) \mathrm{d}i$$

3. Compare model's predicted outcome distributions with actual

$$\begin{aligned} \hat{w}_t[i] &= w_t[0] + G_t \frac{\lambda}{\lambda + 1 - B} \int_0^i \hat{a}'[j]\hat{b}[j]\mathrm{d}j \\ \hat{v}_t[i] &= v_t[0] + G_t \int_0^i \left(\frac{\hat{a}[j]\hat{b}'[j]}{1-B} + \frac{\hat{a}'[j]\hat{b}[j]}{\lambda + 1 - B} \right) \mathrm{d}j \end{aligned}$$

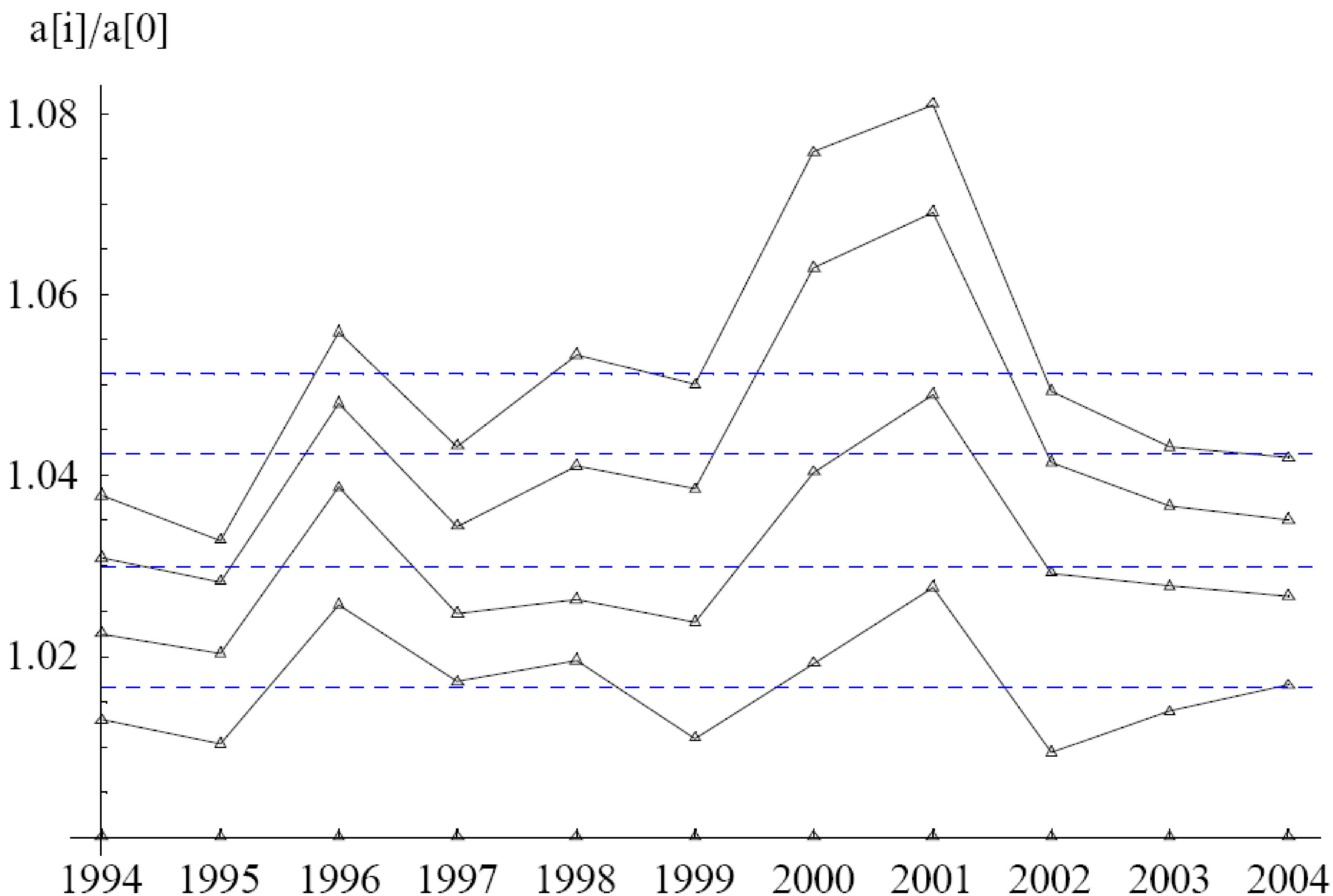


Figure 5. Inferred CEO abilities at 1st, 250th, 500th, and 750th largest firm (relative to 1000th) by year.

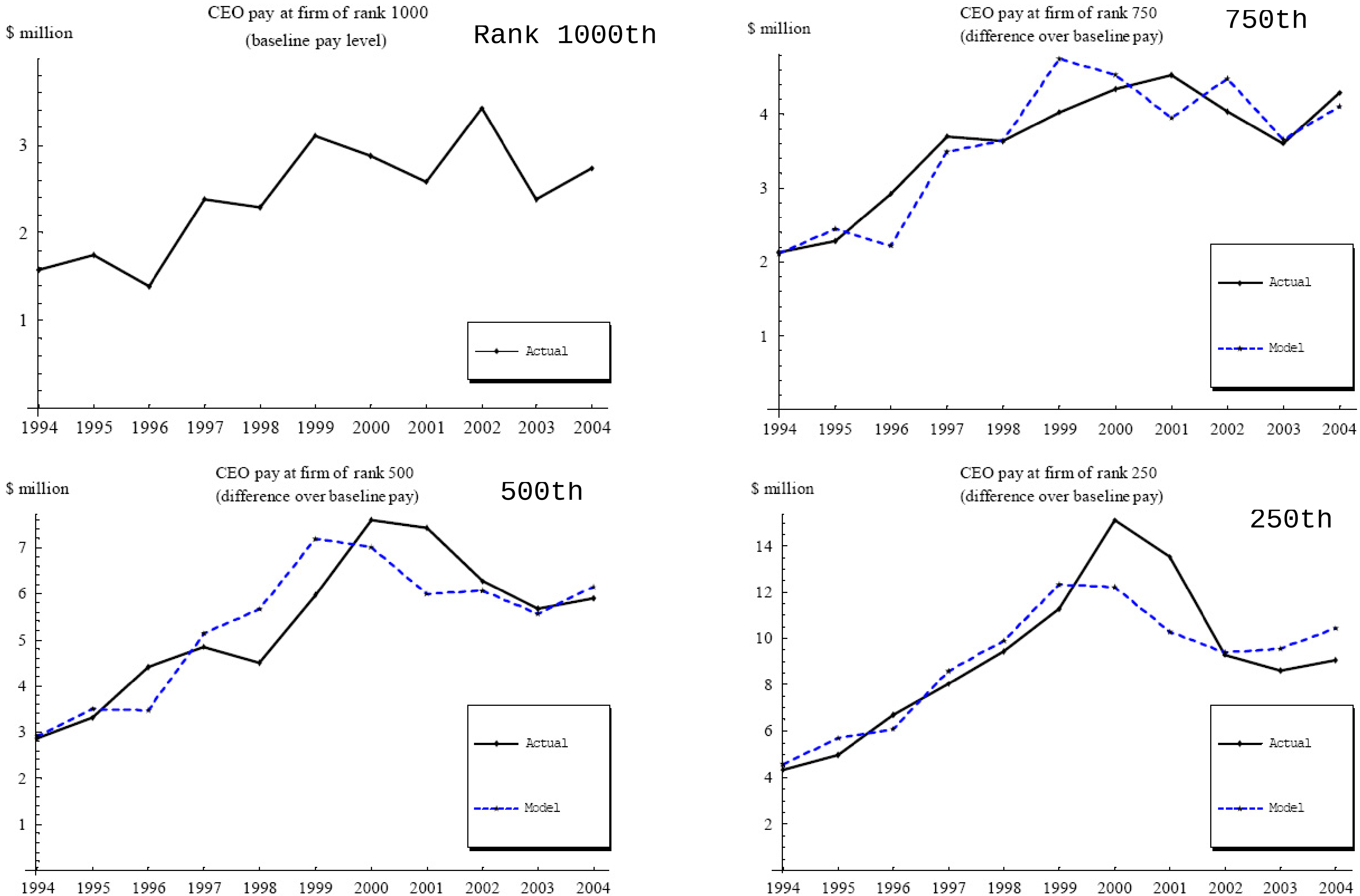


Figure 6a)-d). The difference in pay between the CEOs of selected ranks and the baseline (1000th) CEO.

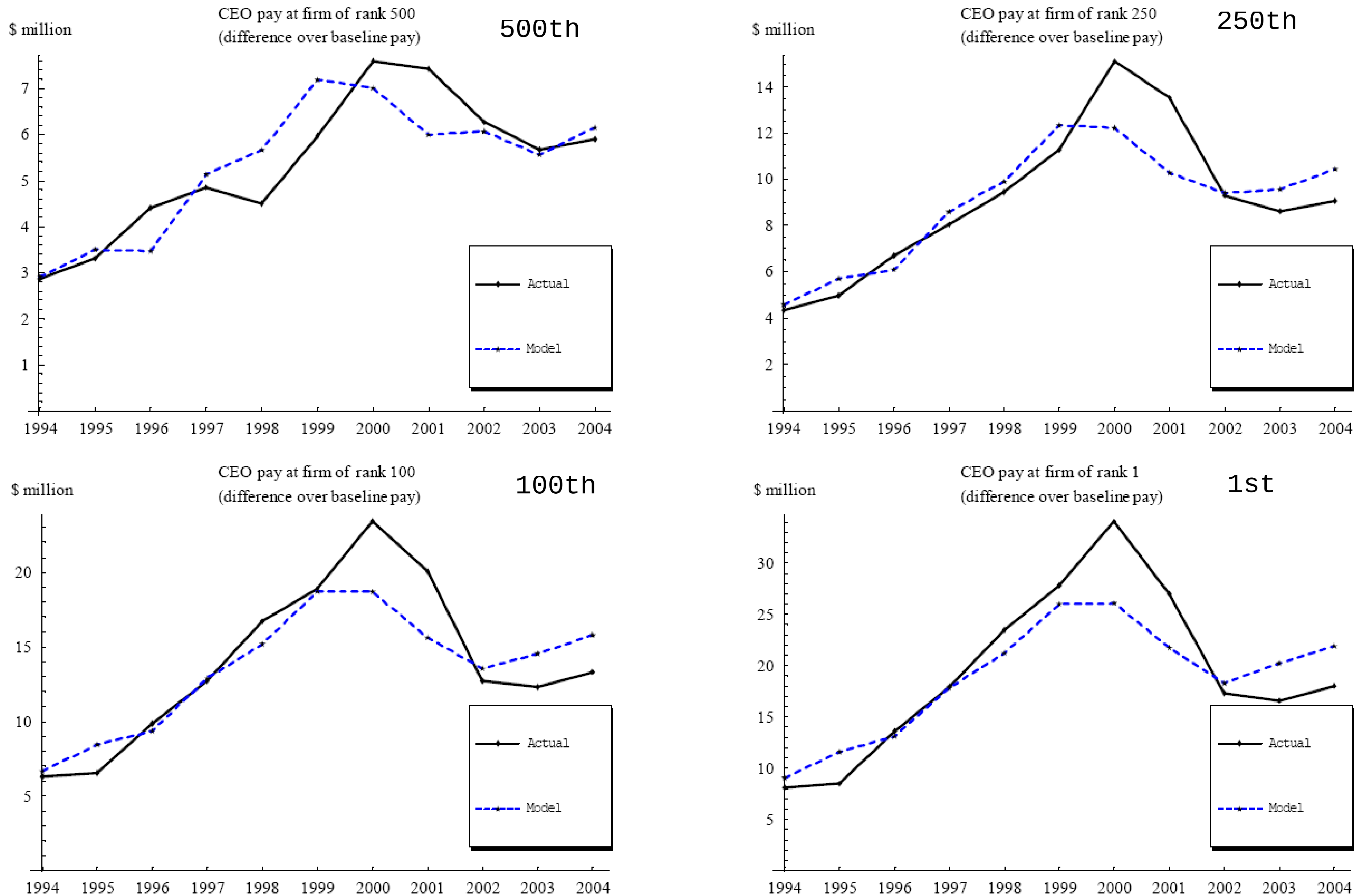


Figure 6c)-f). The difference in pay between the CEOs of selected ranks and the baseline (1000th) CEO.

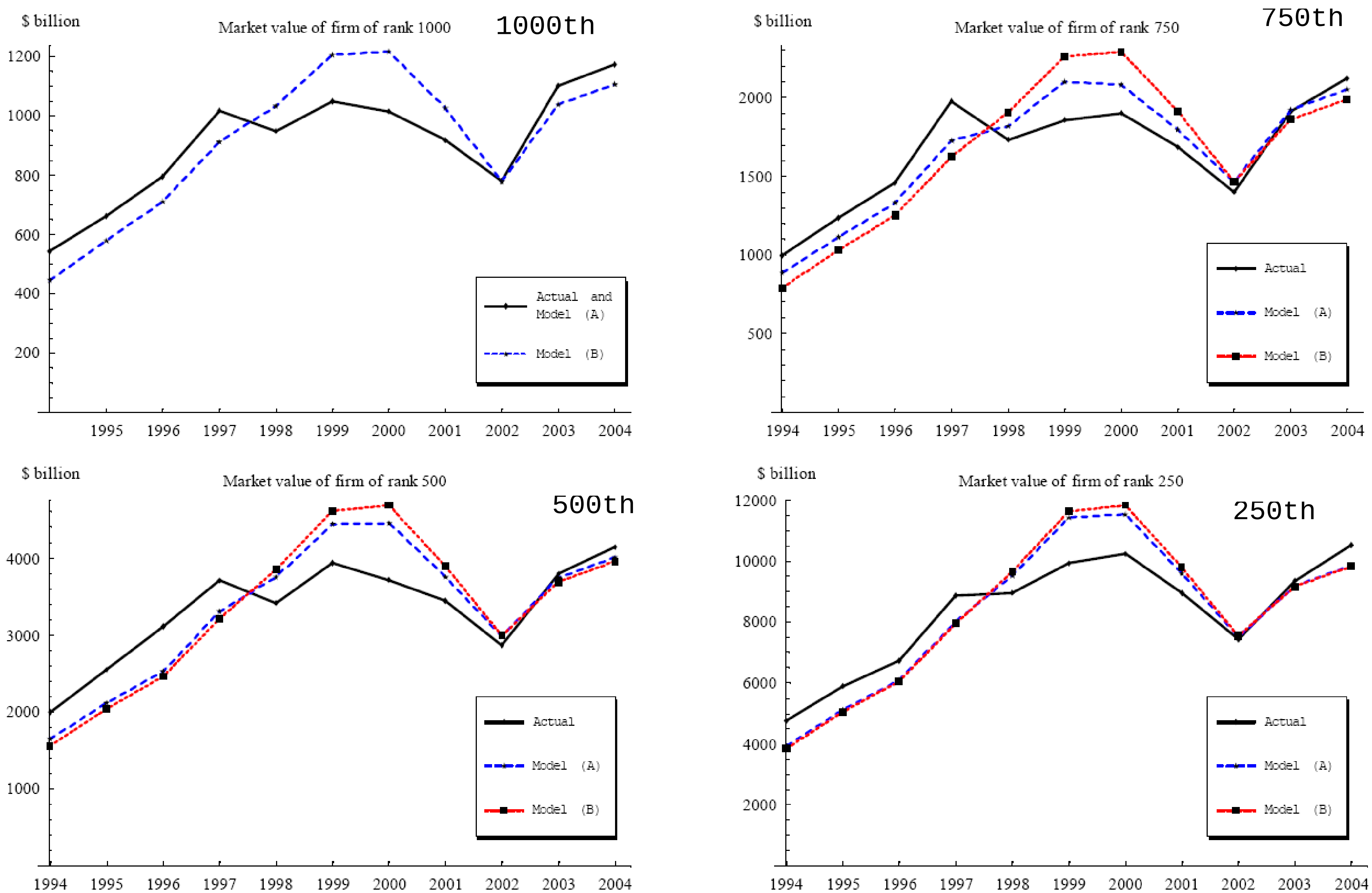


Figure 7a)-d). Predicted market values in the time-invariant calibration.

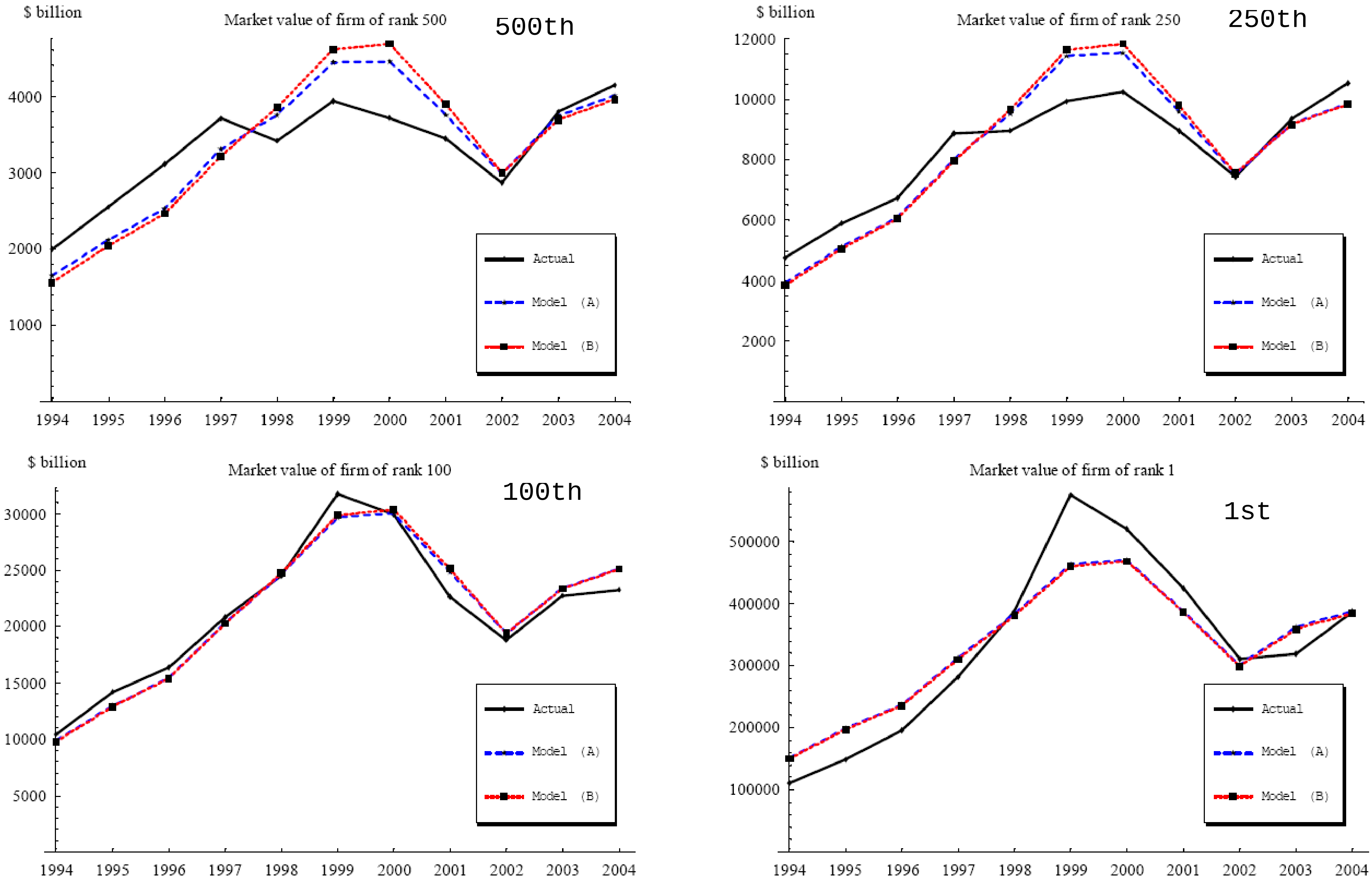


Figure 7c)-f). Predicted market values in the time-invariant calibration.

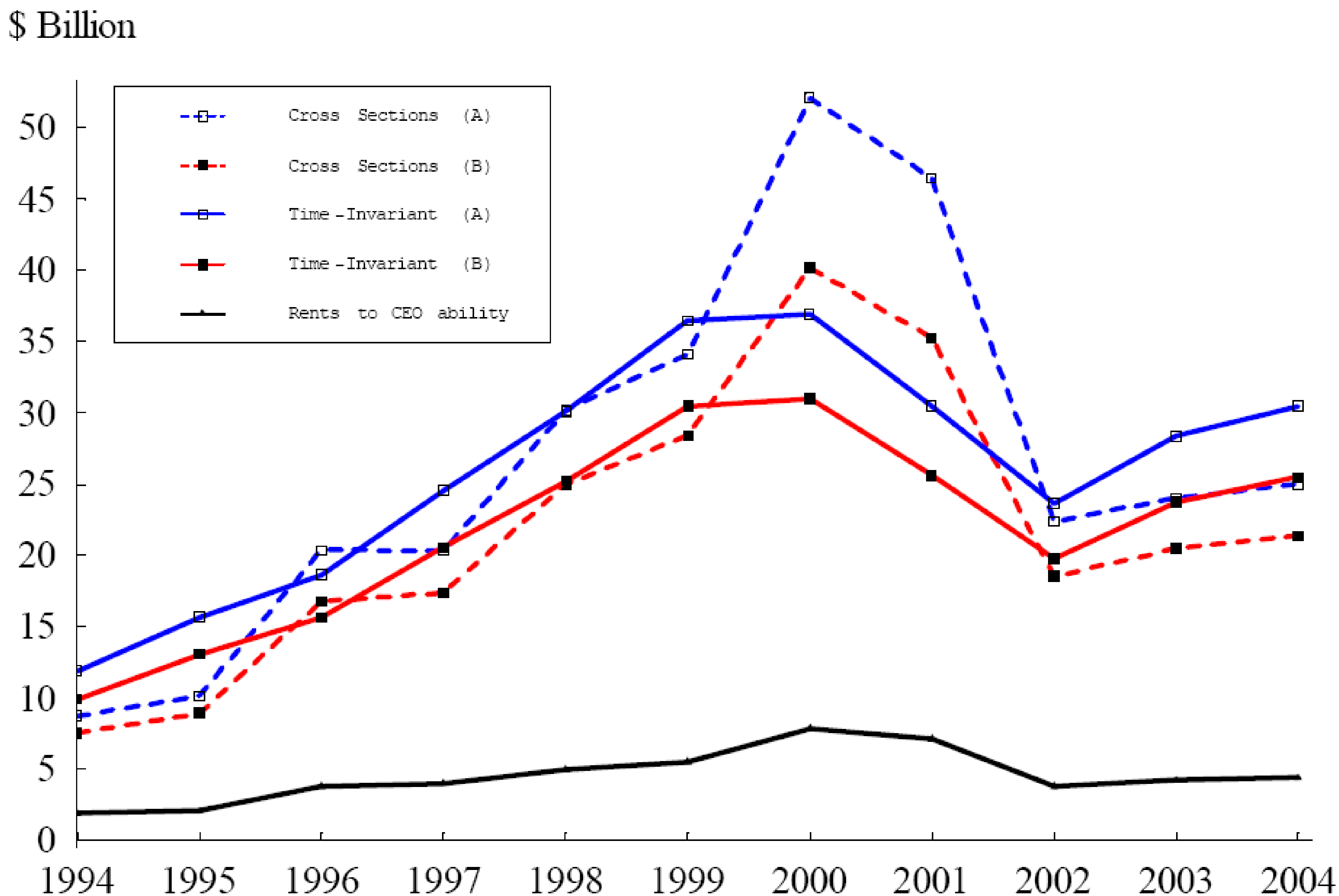


Figure 3. Value of CEO ability and rents to CEOs relative to baseline ability, at the 1000 largest firms.

Conclusion

Assuming a perfectly competitive matching market for CEOs and firms…

- Observed relation of CEO pay and firm size can be explained by competition for small differences in talent
- Value of scarce CEO ability of 1000 largest firms in 2004 $21-25Bn
- Time variation in distribution of CEO pay consistent with time-invariant distribution of unobservables---except during 2000-01
- Assignment model is a fruitful way of modeling CEO pay levels
- How much of pay levels explained by competitive forces and how much by market failures is still an open question.