Common Value Auctions with Costly Entry

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Abstract

We consider a model where potential bidders consider paying an entry cost to participate in an auction. The value of the object sold depends on an unknown state of the world, and the bidders have conditionally i.i.d. signals on the state. We consider mostly the case where entry decisions are taken after observing the signal. We compare first- and second-price auction formats and show that for many symmetric equilibria of the game, first-price auction results in higher expected revenue to the seller.

1 Introduction

Bidding in an auction is often costly. At the very least, each bidder loses the opportunity cost of time spent in preparing the bid and paying attention to the eventual outcome. When the object for sale is valuable and information is dispersed among potential bidders, these costs can be substantial.

We consider a setting where a large number of potential bidders have observed a signal on its true value. The value of the object is common to all bidders, but the bidders are differentially informed about the true value. For the most part in this paper, we consider the case where the true value of the object depends on a binary random variable $\omega \in \{0, 1\}$ (state of the world), and we also assume that the signals take binary values $\theta^i \in \{\theta^H, \theta^L\}$.

At the beginning of the game, each potential bidder decides whether to enter the auction at a positive cost $c > 0$. We consider the case where a single object is for sale (and discuss extensions to the case of a fixed number of objects). Furthermore, we assume anonymity on the part of the bidders so that entry involves a coordination problem. Entry can be profitable only when a limited number of other bidders enter.

We analyze the symmetric equilibria of the entry game under first-price and second-price auction rules. Not surprisingly, equilibrium entry decisions are in mixed strategies for both types of auctions. Conditional on entry, optimal bidding strategies are qualitatively quite different. In the first-price auction, equilibrium bids are mixed strategies both for bidders with signal θ^H and those with signal θ^L. In the second-price auction, bidding conditional on the more pessimistic signal θ^L is in pure strategies, but bidding conditional on θ^H is in mixed strategies when there are many potential bidders. Only in the case with just two potential bidders, the bids of both types of entrants are in pure strategies in the second price auction.

Our main result is that in contrast to most findings in common value auctions, the first-price auction often dominates the second-price auction in terms of the expected revenue to the seller. To understand this result, it is useful to consider the entry decisions of a social planner under the constraint of symmetric strategies (i.e. conditional on the signal, bidders use identical entry strategies). Since the model is one of common values, allocative efficiency is not an issue, and as a result, the planner maximizes the probability of allocating the object weighted by its value in the two states subject to paying the entry costs.

The planner gains from adding a bidder only when no other bidders are present. In a second-price auction, bidders with high signals make a positive profit in the auction if there are no other bidders of high type. If she is the only bidder in the auction, the high type bidder receives her entire marginal contribution to the social welfare. If bidders with low signals are present, she pays a positive price but still makes a positive expected profit due to a more optimistic signal. Hence the incentives to enter are stronger than socially optimal for the bidders with high signals.

In a first-price auction, each bidder pays her own bid. In any symmetric equilibrium, an entering bidder must believe that there is a positive probability that no other bidders enter. This leads to mixed strategy equilibria in the bidding stage, and hence also to potential complications in evaluating the expected payoffs of the two types of bidders. While it is clear that the bidders with low signals must have zero bids in the support of their bid distribution, it is more surprising that for some parameter values this is also the case for the bidders with high signals. Since there are no mass points in the bid distributions (by standard arguments), this implies that both types of bidders are willing to place bids that win at zero price if and only if no other bidder has entered. Hence the expected payoff to both types of bidders coincides with their marginal contribution to social welfare, and as a result equilibrium entry is at socially optimal level.

Since entry decisions are in mixed strategies, entering bidders must make a zero expected payoff. This implies that the seller gets the entire expected social surplus in revenue. Since the entry decisions in the first-price auction are expected surplus maximizing, the seller must gain relative to the second-price auction.

It is also possible that zero bids are not in the support of the bidding strategies for bidders with high signals. For the case of two potential bidders, we show that the first-price auction still dominates the second price auction in terms of expected revenue. For a large number of potential bidders, we show that for small entry costs this ranking can be reversed. For high entry costs, the first-price auction yields a higher revenue in this case as well.

Two additional features of the symmetric second-price equilibria with many potential bidders deserve special mention. The equilibrium bid conditional on a low signal is not uniquely determined and the bids conditional on high signal are mixed. In order to understand these results, it is useful to recall winners curse for common value auctions. As usual, the equilibrium bid in a second price auction is given by the expected value of the object conditional on having the highest bid, conditional on tying the second highest bid and conditional on winning the object. Suppose that all the high type bidders submit the same bid. With uniform rationing, the probability of winning is the highest when the number of tying bids is the smallest. Since the signals are affiliated with the true value of the object, this is bad news on the expected value. By deviating to a slightly higher price, a bidder with a high signal wins in all cases including those in which there are large numbers of other bidders. Hence a pure strategy equilibrium cannot exist.

Consider next the incentives of the low type bidders. In a second price auction, they can win only in the cases where no bidders with high signal have entered. If all low type bidders pool at the same bid, winning is more likely when there are fewer other bidders with low signals. Again by affiliated values, this is positive news on the value of the object. By deviating to a slightly higher price, a low type bidder wins the object in all cases with no bidders with high signals present. In a sense, low bidders experience a winner's blessing at the pooled bid and as a consequence, a continuum of pooled equilibrium prices exists.

In the last section of this paper, we outline some generalizations of our findings. First of all, the revenue ranking favoring the first-price auction remains true for some parameter values of the model even if the entry decision is taken at the ex ante stage, i.e. before knowing own signal realization. At this moment, we are not sure how often this happens (relative to the interim case that we mostly study). Randomized entry decisions give a strong incentive for placing a zero bid in the first price auction if it is likely that no competition is present. If bidders with both types of signals place such a bid, then the timing of information revelation is immaterial. We also discuss the extension to settings with multiple signals and also the case where many identical objects are for sale.

We should stress that we are not claiming a general result showing the uniform superiority of first-price auction when participation in the auction is random. As the entry cost $c \to 0$, the usual reasons based on linkage principal become relatively more important, and second-price auctions are likely to dominate. Our goal is to demonstrate that in a non-trivial set of models, limited and random entry may reverse the usual revenue comparisons.

1.1 Related Literature

Auctions with endogenous entry have been modeled in two separate frameworks. In the first, entry decisions are taken at an ex ante stage where all bidders are identical. Potential bidders learn their private information only upon paying the entry cost. Hence these models can be though of as games with endogenous information acquisition. French & McCormick (1984) gives the first analysis of an auction with an entry fee in the IPV case. Harstad (1990) and Levin & Smith (1994) analyze the affiliated values case. These papers show that due to business stealing, entry is excessive relative to social optimum. They also show that second-price auctions results in higher expected revenues than the first-price auction.

In the other strand, bidders decide on entry only after knowing their own signals. Samuleson (1985) and Stegeman (1996) are early papers in the IPV setting where this question has been taken up. Due to revenue equivalence in the IPV case, comparisons across auction formats are not very interesting. To the best of our knowledge, common values auctions have not been analyzed in this setting. Hence our paper is the first to ask how the auction format affects information aggregation through entry.

Finally some recent papers have analyzed common values auctions with some similarities to our paper. Lauermann & Wolinsky (2013) analyze first-price auctions where an informed seller chooses the number of bidders to invite to an auction. In their setting, it is also important to account for the winner effect when computing the expected value of the object. Atakan & Ekmekci (2014) consider a common value auction where the winner in the auction has to take an additional action after winning the auction. This leads to a non-monotonicity in the value of winning the auction that has some resemblance to the forces in our model that lead to non-monotonic entry (i.e. bidders with both types of signals enter with positive probability).

2 Binary model

We start by laying out the basic model. The state of the world is a binary random variable $\omega \in \{0, 1\}$ with a prior probability

$$q = \Pr\{\omega = 1\}.$$

The common value of the object in state ω is $v(\omega)$ for all the bidders, and we assume that $v(1) > v(0) > 0$.

At the outset, each potential bidder i observes a binary signal $\theta_i \in \{\theta^h, \theta^l\}$. Let $\Pr\left(\theta = \theta^l \left|\omega = 0\right.\right) = \Pr\left(\theta = \theta^h \left|\omega = 1\right.\right) := \alpha > 1/2$ and denote by $q^h := \Pr\left(\omega = 1 \left|\theta = \theta^h\right.\right)$ and $q^l := \Pr\left(\omega = 1 \left|\theta = \theta^l\right.\right)$ the posterior based on a high and low signal, respectively. For example, if prior is $q = 1/2$, then $q^l = 1 - \alpha$ and $q^h = \alpha$. The signals are assumed to be i.i.d. conditional on the state of the world.[1]

After observing θ_i, each player i decides whether to pay an entry cost $c > 0$ with $c < v(0)$ and submit a bid b_i in an auction for a single object or whether to stay out and receive a certain payoff of 0. At the moment of bidding, i does not know how many other bidders have chosen to participate in the auction. Furthermore, we distinguish between two alternative auction formats: the first-price auction (FPA) and the second-price auction (SPA).

The entry strategy of potential bidder i specifies the probability of entry:

$$\pi_i : \{\theta^h, \theta^l\} \to [0, 1].$$

We use π_i^h for $\pi_i\left(\theta^h\right)$ and π_i^l for $\pi_i\left(\theta^l\right)$. Since we concentrate on symmetric equilibria, we often omit subscripts. Similarly a bid strategy is a function

$$b_i : \{\theta^h, \theta^l\} \to \Delta\left(\mathbb{R}_+\right),$$

where we have allowed for randomized bids.

The bidders are risk neutral and bid to maximize their expected profit from the auction.

3 A two-player version

We start the analysis with the simple case where there are only two potential bidders. There are now just two players, and we look for a symmetric mixed strategy equilibrium. We are interested in seeing how the equilibrium entry decisions depend on the format of the auction chosen.

[1] We consider the symmetric case with $\Pr\{\theta = \theta^h \left|\omega = 1\right.\} = \Pr\{\theta = \theta^l \left|\omega = 0\right.\}$ for notational simplicity. The results in this paper go through with asymmetric specifications too.

We start by characterizing the efficient solution, i.e. the symmetric entry probabilities that a utilitarian planner would choose. The planner's problem is to

$$\max_{\pi^h,\pi^l} V\left(\pi^h,\pi^l\right) :=$$
$$q\left[\left(1-\left(1-\alpha\pi^h-(1-\alpha)\,\pi^l\right)^2\right)v\,(1)-2\left(\alpha\pi^h+(1-\alpha)\,\pi^l\right)c\right]$$
$$+\,(1-q)\left[\left(1-\left(1-(1-\alpha)\,\pi^h-\alpha\pi^l\right)^2\right)v\,(0)-2\left(\alpha\pi^l+(1-\alpha)\,\pi^h\right)c\right].$$

From the first order conditions to this concave maximization problem, we get:

$$\begin{aligned}
\widehat{\pi}^h &= \frac{1}{2\alpha-1}\left(\alpha\frac{v\,(1)-c}{v\,(1)}-(1-\alpha)\,\frac{v\,(0)-c}{v\,(0)}\right),\\
\widehat{\pi}^l &= \frac{1}{2\alpha-1}\left(\alpha\frac{v\,(0)-c}{v\,(0)}-(1-\alpha)\,\frac{v\,(1)-c}{v\,(1)}\right).
\end{aligned}$$

This is a valid solution if $\widehat{\pi}^l \geq 0$ and $\widehat{\pi}^h \leq 1$. These restrictions are satisfied if

$$\alpha\frac{v\,(0)-c}{v\,(0)} \geq (1-\alpha)\,\frac{v\,(1)-c}{v\,(1)}$$

and

$$\frac{\alpha}{1-\alpha} \geq \frac{v\,(1)}{v\,(0)}.$$

If the first inequality is violated, then only high signal players enter in the efficient solution. We shall see that in this case competitive entry followed by the SPA and the FPA also result in efficient entry levels and identical expected revenues to the seller. In the case of an interior solution, we shall see that FPA dominates SPA in terms of expected revenue. Notice that the second inequality restriction is needed only because here the number of potential entrants is limited to two. In the model with a large number of potential bidders a corner solution $\widehat{\pi}^h = 1$ would be very costly to the planner.

Lemma 1 *Fix $\pi^h \in (0,1)$ in the planner's problem and let $\widehat{\pi}^l\left(\pi^h\right)$ be the conditionally optimal entry rate by the low type bidders. Then $\widehat{\pi}^l\left(\pi^h\right)$ is decreasing in π^h and $V\left(\pi^h,\widehat{\pi}^l\left(\pi^h\right)\right)$ is strictly concave in π^h.*

Proof. By straightforward computation. ■

3.1 Entry game with second-price auction

With only two bidders, the realized price is the lower of the two bids if both players enter, and zero if only one player enters. Equilibrium inference about the value of the good is straightforward. As usual in common value auctions, the optimal bid is obtained by assuming that both bidders have submitted the same bid. It is clear that this can occur only if the bidders have observed the same signals. Hence, if both types enter with positive probability in equilibrium, we can write

$$\begin{aligned} p^h &= \mathbb{E}_\omega[v(\omega) \,|\theta_1 = \theta_2 = \theta^h], \\ p^l &= \mathbb{E}_\omega[v(\omega) \,|\theta_1 = \theta_2 = \theta^l]. \end{aligned}$$

The bidding strategies of both types are thus pure.

If only high type bidders enter in equilibrium, then their bids remain as above and low type bidders bid any amount below b^h in any sequentially rational continuation following a deviation since they want to win only conditional on having entered alone. Since the expected profit to the deviating low type bidder is exactly her contribution to the expected social surplus, she will choose to stay out of the market if high types enter efficiently whenever the planner's solution is not an interior solution. Notice that high signal bidders also collect exactly their expected marginal contribution at the efficient profile. We conclude that efficient entry is an equilibrium in the entry game followed by second-price auction as long as $\widehat{\pi}^l = 0$ in the efficient solution.

Suppose next that $\widehat{\pi}^l > 0$. In this case, the social planner gains from a high type entry if and only if there are no other bidders present in the market. In the SPA, high type bidders gain their social contribution if there are no other bidders present, but in addition, they also make a positive profit whenever another bidder with low signal has entered. Hence, with $\widehat{\pi}^l > 0$, the expected private profit of a high signal type exceeds the social benefit due to a business stealing effect, and equilibrium level of π^h exceeds the conditional social optimum (i.e. optimum given π^l). Since the low type bidders make a positive profit only when there are no other bidders present, their private profit coincides with their marginal social contribution. Since the conditionally efficient level of $\pi^l\left(\pi^h\right)$ is decreasing in π^h, we conclude that:

Proposition 1 *Suppose* $\widehat{\pi}^l > 0$. *Then the entry equilibrium* $\left(\pi_S^h, \pi_S^l\right)$ *followed by a second-price auction is characterized by: i)* $\pi_S^h > \widehat{\pi}^h$ *and ii)* $\pi_S^l < \widehat{\pi}^l$.

3.2 Entry game with first-price auction

Consider next the case of a first-price auction. Using standard arguments, we can show that symmetric equilibria in this case must be in atomless mixed strategies. Denote by $\pi^s > 0$ the equilibrium entry probability and by $b^s(p)$ the equilibrium bid distribution for $s \in \{h, l\}$. We can then let $\pi^s(p) = \pi^s - b(p)$ denote the probability that i has entered the auction and placed a bid above level p, conditional on having observed signal s. Let supp $\pi^s(\cdot)$ denote the support of the bid distribution b^s.

A first simple observation is that if both types of potential bidders enter with a positive probability and a bid of 0 is in the support of their bid distributions, then entry must be at efficient level. Since there are no atoms, a bid of zero wins only if there are no other active bidders. If this bid is in the support of both types, we conclude that potential bidders of both types earn an expected profit exactly equal to their marginal contribution and hence entry must be at the constrained efficient level. We consider next the case where 0 is not necessarily in the support of the bid distributions.

Suppose that both types of bidders enter with positive probability and that a bid p is in the support of the bid strategies of both types of potential bidders. Since both must be indifferent between entering and not, and since the two types of bidders have different assessments of the relative probabilities of the two states, we must have indifference between entering with bid p and staying out across the two states. In other words, the following must hold:

$$\begin{aligned} & \left(1-\alpha\pi^h(p)-(1-\alpha)\pi^l(p)\right)(v(1)-p) \\ = & \left(1-\alpha\pi^l(p)-(1-\alpha)\pi^h(p)\right)(v(0)-p)=c, \end{aligned}$$

which leads to

$$\pi^{h}(p) = \frac{1}{2\alpha-1}\left(\alpha\frac{v(1)-p-c}{v(1)-p}-(1-\alpha)\frac{v(0)-p-c}{v(0)-p}\right), \tag{1}$$
$$\pi^{l}(p) = \frac{1}{2\alpha-1}\left(\alpha\frac{v(0)-p-c}{v(0)-p}-(1-\alpha)\frac{v(1)-p-c}{v(1)-p}\right).$$

This observation leads to the following lemma that is key to all our revenue comparisons.

Lemma 2 *Suppose that $0<\pi^{l}<\pi^{h}<0$. Then either i) $0\in$ supp $\pi^{s}(\cdot)$ for $s\in\{h,l\}$ or ii) supp $\pi^{l}(\cdot)=\left[0,p^{l}\right]$ and supp $\pi^{h}(\cdot)=\left[p^{l},p^{h}\right]$ for some $0<p^{l}<p^{h}<v(1)$.*

Proof. First of all, it is obvious that $0\in\left\{\text{supp }\pi^{l}(\cdot)\cup\text{ supp }\pi^{h}(\cdot)\right\}$ since otherwise the bidder submitting the lowest bid would have a profitable deviation. We prove the claim by showing that whenever $0\notin$ supp $\pi^{h}(\cdot)$, the supports are non-overlapping intervals with $0\in$ supp $\pi^{l}(\cdot)$. To show this, we notice that if the supports overlap on a non-empty interval, then equations (1) hold over this interval, and we can differentiate the equations to get:

$$\frac{d\pi^{h}(p)}{dp} = \frac{1}{2\alpha-1}\left(-\frac{\alpha c}{(v(1)-p)^{2}}+\frac{(1-\alpha)c}{(v(0)-p)^{2}}\right),$$
$$\frac{d\pi^{l}(p)}{dp} = \frac{1}{2\alpha-1}\left(-\frac{\alpha c}{(v(0)-p)^{2}}+\frac{(1-\alpha)c}{(v(1)-p)^{2}}\right).$$

Differentiating for a second time yields:

$$\frac{d^{2}\pi^{h}(p)}{dp^{2}} = \frac{2}{2\alpha-1}\left(-\frac{\alpha c}{(v(1)-p)^{3}}+\frac{(1-\alpha)c}{(v(0)-p)^{3}}\right), \tag{2}$$
$$\frac{d^{2}\pi^{l}(p)}{dp^{2}} = \frac{2}{2\alpha-1}\left(-\frac{\alpha c}{(v(0)-p)^{3}}+\frac{(1-\alpha)c}{(v(1)-p)^{3}}\right)<0.$$

We see here that

$$\frac{d\pi^{l}(p)}{dp}<0 \text{ for all } p,$$

and that the following implication holds:

$$\frac{d\pi^{h}(p)}{dp}\geq 0\Longrightarrow\frac{d^{2}\pi^{l}(p)}{dp^{2}}>0.$$

The first property implies immediately that if bid p is in the support of $\pi^l(\cdot)$, then so is $p' < p$ demonstrating $0 \in \text{supp } \pi^l(\cdot)$.

Since the conclusion of the second implication is false by the second line in equation (2), we know that its premise must be false as well and hence $\frac{d\pi^h(p)}{dp} < 0$. By the same logic as above, $0 \in \text{supp } \pi^h(\cdot)$. Hence we conclude that either $0 \in \text{supp } \pi^h(\cdot)$ or the supports do not overlap.

Gaps in $\text{supp } \pi^l(\cdot) \cup \text{supp } \pi^h(\cdot)$ and mass points in either distribution are ruled out by usual arguments. Notice also that by the second line in equation (1) for $p' < p$

$$\{p', p\} \subset \{\text{supp } \pi^l(\cdot) \cap \text{ supp } \pi^h(\cdot)\} \Rightarrow \pi^l(p') < \pi^l(p).$$

This rules out the possibility that $\text{supp } \pi^l(\cdot)$ is a union of disjoint intervals.

Finally, we need to show that in the case of non-overlapping interval supports $0 \in\text{supp } \pi^l(\cdot)$. Let $p^{\max}$ be the maximal bid in the union of the two supports. Since we have assumed that $0 < \pi^l < \pi^h < 0$, we have

$$\mathbb{E}\left[v(\omega) - p^{\max}|\theta^h\right] > \mathbb{E}\left[v(\omega) - p^{\max}|\theta^l\right].$$

Hence $p^{\max} \in\text{supp } \pi^h(p)$,and hence also $0 \in\text{supp } \pi^l(\cdot)$. ■

Lemma 3 *If the equilibrium entry rates in the first-price equilibrium* $\left(\pi^h, \pi^l\right) \neq \left(\widehat{\pi}^h, \widehat{\pi}^l\right)$, *then*

$$\pi^h > \widehat{\pi}^h \text{ and } \pi^l < \widehat{\pi}^l.$$

Proof. Against any fixed rate π^l of entry for the low types, the equilibrium rate of entry is at least $\widehat{\pi}^h\left(\pi^l\right)$. For a lower rate, entry followed by $p = 0$ would yield a profitable deviation. The conclusion of the lemma follows immediately. ■

With the help of these lemmas, we are ready to prove the main result of this section.

Theorem 1 *In the two-bidder model, the first-price auction yields always a higher expected revenue than the second price auction.*

Proof. i) If $0 \in \text{supp } \pi^h(\cdot) \cap \text{supp } \pi^l(\cdot)$, then entry rates $\left(\widehat{\pi}^h, \widehat{\pi}^l\right) \in$ are consistent with the First-Price Auction. Since $\left(p^h, p^l\right) = (0,0)$ is in the support of the bidders, their expected payoff coincides with the expected payoff of the planner. Since the bidders' expected payoffs in this equilibrium are zero, the revenue in the auction coincides with the maximal expected social surplus. Proposition 1 shows that in the second-price auction, entry rates differ from the planner's solution. Since the bidders' expected payoffs are non-negative in any equilibrium, the claim follows from the strict concavity of the planner's optimization problem.

ii) If $0 \notin \text{supp } \pi^h(\cdot)$ in the equilibrium of the game with first-price auction. Then, by the previous lemma, $\text{supp } \pi^l(\cdot) = \left[0, p^l\right]$ and $\text{supp } \pi^h(\cdot) = \left[p^l, p^h\right]$ in the first-price auction.

Let $\left(\pi^h, \pi^l\right)$ denote the equilibrium entry rates for the game with first-price auction bidding. Keep entry rates fixed at $\left(\pi^h, \pi^l\right)$, but switch the auction format to second-price auction. Bidder with θ^l is still indifferent between entering and not because in first-price auction by bidding 0 she gets the same allocation and price as in the second price auction. On the other hand, p^l is also in $\text{supp } \pi^l(p)$. At that price, she gets the good with probability 1 if and only if there is no opponent of type θ^h. Denote this event by A.

Thus, consider the allocation rule where a bidder gets the good if and only if event A happens. Compare two different pricing rules for event A as follows: i) pay p^l for sure, and ii) pay $p = 0$ if no low type presents, and otherwise pay $p = \mathbb{E}\left(v|\theta = \theta^l, \text{a low type present}\right)$. A bidder of type θ^l is indifferent between these two situations since i) is her outcome in first price auction for bid $b = 0$, which is within her bidding support, and ii) is her outcome in second price auction (if she outbids equilibrium bid by an infinitesimal ε). But then a bidder type θ^h prefers case ii) to i), because she finds it more likely that $p = 0$ than bidder type θ^l. Since $p^l \in \text{supp } \pi^h(\cdot)$ in the first-price auction, her expected profit in that auction must be c. Hence, the high type makes a strictly higher profit in the second price auction. Hence, there must be more entry by bidders of type θ^h in the second-price auction if $\pi^h < 1$, and if $\pi^h = 1$, then the rent to θ^h is higher in the second-price auction. By Lemma 1 and Lemma 3, we conclude that the second-price auction yields a lower social surplus and hence also lower expected revenue to the seller. ■

4 Many potential bidders

We consider the limiting model where the number of potential bidders $N \to \infty$. By usual arguments, given a sequence of symmetric entry strategies for finite games $\{\pi_N^s\}$, $s = l, h$, such that $N \cdot \pi_N^s \to \pi^s$, the realized number of entering agents converges to a Poisson random variable with parameter $\alpha\pi^h + (1-\alpha)\pi^l$ in state $\omega = 1$ and with parameter $(1-\alpha)\pi^h + \alpha\pi^l$ in state $\omega = 0$. Hence, we will be looking for equilibrium "entry intensities" π^l and π^h.

As in the previous section, we start by considering the social planner's problem: She chooses entry intensities π^l and π^h to maximize social surplus net of entry cost. The objective function of the social planner is given by:

$$\begin{aligned} W\left(\pi^l, \pi^h\right) &= q\left[\left(1 - e^{-\alpha\pi^h - (1-\alpha)\pi^l}\right) v(1) - \left(\alpha\pi^h + (1-\alpha)\pi^l\right) c\right] \\ &\quad + (1-q)\left[\left(1 - e^{-(1-\alpha)\pi^h - \alpha\pi^l}\right) v(0) - \left((1-\alpha)\pi^h + \alpha\pi^l\right) c\right]. \end{aligned}$$

The first term in square brackets computes the benefit and cost of entry if the state is high and the second corresponds to the low state. To simplify the formulas slightly, write

$$\begin{aligned} \lambda(1) &= \alpha\pi^h + (1-\alpha)\pi^l, \\ \lambda(0) &= (1-\alpha)\pi^h + \alpha\pi^l, \end{aligned}$$

for the Poisson parameter conditional on the state of the world.

This is a concave problem with first-order conditions for interior solutions given by:

$$\begin{aligned} q\alpha\left[e^{-\lambda(1)}v(1) - c\right] + (1-q)(1-\alpha)\left[e^{-\lambda(0)}v(0) - c\right] &= 0, \\ q(1-\alpha)\left[e^{-\lambda(1)}v(1) - c\right] + (1-q)\alpha\left[e^{-\lambda(0)}v(0) - c\right] &= 0. \end{aligned}$$

This is satisfied when

$$\begin{aligned} e^{-\lambda(1)}v(1) &= c, \\ e^{-\lambda(0)}v(0) &= c, \end{aligned}$$

or

$$\widehat{\pi}^h = \frac{1}{2\alpha - 1}\left(\alpha \log\left(\frac{v(1)}{c}\right) - (1-\alpha)\log\left(\frac{v(0)}{c}\right)\right),$$
$$\widehat{\pi}^l = \frac{1}{2\alpha - 1}\left(-(1-\alpha)\log\left(\frac{v(1)}{c}\right) + \alpha\log\left(\frac{v(0)}{c}\right)\right).$$

For this to yield a valid solution, we must have $\widehat{\pi}^l > 0$, so our assumption in terms of model parameters is:

$$\alpha \log\left(\frac{v(0)}{c}\right) > (1-\alpha)\log\left(\frac{v(1)}{c}\right)$$

or

$$\frac{\alpha}{1-\alpha} > \frac{\log(v(1)) - \log(c)}{\log(v(0)) - \log(c)}.$$

Notice that as $c \to 0$, the right hand side converges to 1. Therefore, we always have $\pi^l > 0$ for low enough c. On the other hand, by increasing c towards $v(0)$, at some point $\widehat{\pi}^l$ reduces to zero and we get a corner solution where only high types enter.

Notice also that we have immediately the result that $\widehat{\pi}^h > \widehat{\pi}^l$ since

$$\widehat{\pi}^h - \widehat{\pi}^l = \frac{1}{2\alpha - 1}\log\left(\frac{v(1)}{v(0)}\right).$$

4.1 Second-price auction with unobserved entry

With more than two potential bidders, the effect of conditioning upon winning the auction is more complicated than in the standard case of a fixed number of bidders with a continuum of signal. In the standard case, the conditioning event is that the winner's bid ties with the highest bid amongst other bidders. Under increasing strategies this again translates into having a tie between the two highest signals.

In our case, winning the auction gives also information about the number of other entrants to the auction. In accordance with our interest in symmetric equilibria, we assume symmetric rationing in case of tied bids. This implies that a bidder is more likely to win the object if there are fewer bidders. This in turn gives information on the value of the object for sale if entry takes place at different rates for different signals.

It is easy to show that in any symmetric equilibrium of the SPA, bidders with low signals must bid below the bidders with high signals. If high signal bidders bid according to a pure strategy, the probability of winning is $\frac{1}{n^h}$, where n^h is the (random) number of entrants with high signals. Hence winning is evidence of low n^h and by affiliated signals this is also evidence in favor of $\{\omega = 0\}$. By the usual logic of SPA, the equilibrium bid must equal the conditional expected value of the object upon winning. By a small upward deviation, any bidder with a high signal wins the object for sure. Under this conditioning event, the value of the object is strictly larger than when submitting the assumed common equilibrium bid. Hence bidders with high signals cannot use the same pure bid in equilibrium, and we must consider a mixed strategy equilibrium for those bidders.

Bidders with low signals face a different updating. Again, with symmetric rationing winning the object gives evidence of a small number of bidders in the auction. In contrast to the bidders with high signals, this is now good news about the value of the object. This translates into a multiplicity of symmetric pure equilibrium bids for bidders with low signals. By deviating to a higher price (still below the lowest bid of the high signal biders), any low signal bidder wins regardless of the number of other low signal bidders (as long as there are no high type bidders). But the expected value of the object is smaller under the new conditioning event. This 'winner's blessing' effect makes it possible to sustain different pure symmetric equilibrium bids for the bidders with low signals.

It is clear that equilibrium entry with a second-price auction is always distorted if bidders with both signals enter with positive intensity. To see this, assume efficient entry. Then by the same arguments as in the previous section, at least the high type gets more than her contribution to social welfare. It is also possible that low type gets more (in a particular equilibrium). So it is possible that too many bidders of both types enter in equilibrium (similarly to the traditional business stealing effect).

Let us derive next the equilibrium, where for given entry rates, the low signal type bids the highest bid consistent with equilibrium. This is the equilibrium where a low type entrant gains exactly her contribution to social welfare. The bid of a player with $\theta = \theta^h$ when equilibrium entry rates are π^h and π^λ is denoted $b^l\left(\pi^l, \pi^h\right)$.

The entry rate of θ^l conditional on state ω is given by:

$$\lambda^{low}(\omega) = \begin{cases} (1-\alpha)\,\pi^l \text{ for } \omega = 1 \\ \alpha\pi^l \text{ for } \omega = 0 \end{cases}$$

Denote the event that no bidder with θ^h enter by A. The probability of getting the object (for bidder of type θ^l), conditional on state ω and event A is (for simplicity denote $\lambda = \lambda^{low}(\omega)$):

$$\begin{aligned} & \Pr(win\,|\omega \text{ and } A) \\ = \; & 1e^{-\lambda} + \frac{1}{2}\lambda e^{-\lambda} + \frac{1}{3}\frac{\lambda^2}{2!}e^{-\lambda} + \frac{1}{4}\frac{\lambda^3}{3!}e^{-\lambda} + ... \\ = \; & \frac{1}{\lambda}\left(-e^{-\lambda} + e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2}{2!}e^{-\lambda} + \frac{\lambda^3}{3!}e^{-\lambda} + \frac{\lambda^4}{4!}e^{-\lambda} + \right) \\ = \; & \frac{1}{\lambda}\left(1 - e^{-\lambda}\right). \end{aligned}$$

For the right conditioning event for the equilibrium bid, let B denote the event that at least one (other) bidder of type θ^l entered. We compute the probability of winning the auction in event B conditional on the state ω and event A as follows:

$$\begin{aligned} & \Pr(\text{win and } B\,|\omega \text{ and } A) \\ = \; & \frac{1}{\lambda}\left(1 - e^{-\lambda}\right) - e^{-\lambda} = \frac{1}{\lambda}\left(1 - e^{-\lambda} - \lambda e^{-\lambda}\right). \end{aligned}$$

Then we can compute the likelihood ratio on the states for a low type that gets the object at price b^l. This decomposes information into 1) the prior belief, 2) own signal, 3) the event that no high types enter, 4) other low types enter and the player in question wins

$$\begin{aligned} \frac{q^l}{1-q^l} &= \frac{q}{1-q}\frac{1-\alpha}{\alpha}\frac{e^{-\alpha\pi^h}}{e^{-(1-\alpha)\pi^h}}\frac{\frac{1}{(1-\alpha)\pi^l}\left(1 - e^{-(1-\alpha)\pi^l} - (1-\alpha)\,\pi^l e^{-(1-\alpha)\pi^l}\right)}{\frac{1}{\alpha\pi^l}\left(1 - e^{-\alpha\pi^l} - \alpha\pi^l e^{-\alpha\pi^l}\right)} \\ &= \frac{q}{1-q}\frac{e^{-\alpha\pi^h}}{e^{-(1-\alpha)\pi^h}}\frac{1 - e^{-(1-\alpha)\pi^l} - (1-\alpha)\,\pi^l e^{-(1-\alpha)\pi^l}}{1 - e^{-\alpha\pi^l} - \alpha\pi^l e^{-\alpha\pi^l}} \\ &: = L. \end{aligned}$$

Similarly, we can compute the belief ratio for high type who gets the object at price b^l:

$$\frac{q^h}{1-q^h} = \frac{q}{1-q}\frac{\alpha}{1-\alpha}\frac{e^{-\alpha\pi^h}}{e^{-(1-\alpha)\pi^h}}\frac{1-e^{-(1-\alpha)\pi^l}}{1-e^{-\alpha\pi^l}} > L.$$

We compute the bid of θ^l in the equilibrium where she is indifferent between winning the auction and losing it at b^l. This is obviously the highest possible pure strategy equilibrium bid by θ^l and we call it the *highest equilibrium.*

$$b^l\left(\pi^l, \pi^h\right) = \frac{L}{1+L}v(1) + \frac{1}{1+L}v(0). \tag{3}$$

Given $b^l\left(\pi^l, \pi^h\right)$, we can compute the expected payoffs V^l and V^h for θ^l and θ^h respectively. Since we concentrate on the highest equilibrium, the the low type gets positive payoff only if no other bidders present, so V^l is easy to compute:

$$V^l = q^l e^{-\alpha\pi^h-(1-\alpha)\pi^l}v(1) + \left(1-q^l\right)e^{-(1-\alpha)\pi^h-\alpha\pi^l}v(0).$$

The expected payoff of θ^h is:

$$\begin{aligned} V^h \;=\;& q^h e^{-\alpha\pi^h}\left(v(1) - \left(1-e^{-(1-\alpha)\pi^l}\right)b^l\right) \\ &+ \left(1-q^h\right)e^{-(1-\alpha)\pi^h}\left(v(0) - \left(1-e^{-\alpha\pi^l}\right)b^l\right), \end{aligned}$$

where b^l depends on π^l and π^h through equation (3). In equilibrium, π^l and π^h must be such that $V^l = V^h = c$. We do not have closed form solutions for equilibrium π^l and π^h, but it can be shown that $\pi^h > \pi^{*h}$ and $\pi^l < \pi^{*l}$, where the starred rates denote the socially efficient entry rates. Hence we conclude that similar to the two-bidder case, the entry rate of θ^h is distorted upwards and the entry of θ^l is distorted downwards.

4.2 First price auction with unobserved entry

We construct the symmetric equilibrium by considering different price regions. We us use notation $\pi^h(p)$ and $\pi^l(p)$ to denote Poisson entry intensity of high and low type, who bid above p.

4.2.1 Overlapping price intervals

If p is within the equilibrium price support for both players, then it must be the case that both types are indifferent between state realization. Therefore, posteriors q^h and q^l play no role, and we have:

$$\begin{aligned} e^{-\alpha\pi^h(p)-(1-\alpha)\pi^l(p)}\left(v\left(1\right)-p\right) &= c \\ e^{-(1-\alpha)\pi^h(p)-\alpha\pi^l(p)}\left(v\left(0\right)-p\right) &= c \end{aligned}$$

or

$$\begin{aligned} \alpha\pi^h\left(p\right)+\left(1-\alpha\right)\pi^l\left(p\right) &= \log\left(\frac{v\left(1\right)-p}{c}\right) \\ \left(1-\alpha\right)\pi^h\left(p\right)+\alpha\pi^l\left(p\right) &= \log\left(\frac{v\left(0\right)-p}{c}\right) \end{aligned}$$

and we get

$$\pi^h\left(p\right) = \frac{1}{2\alpha-1}\left(\alpha\log\left(\frac{v\left(1\right)-p}{c}\right)-\left(1-\alpha\right)\log\left(\frac{v\left(0\right)-p}{c}\right)\right), \quad (4)$$

$$\pi^l\left(p\right) = \frac{1}{2\alpha-1}\left(-\left(1-\alpha\right)\log\left(\frac{v\left(1\right)-p}{c}\right)+\alpha\log\left(\frac{v\left(0\right)-p}{c}\right)\right). \quad (5)$$

If the supports of $\pi^h\left(\cdot\right)$ and $\pi^l\left(\cdot\right)$ overlap on an interval of positive length, then within this interval, the above equalities hold for all p, and we can differentiate the entry intensities to get our first results:

$$\begin{aligned} \pi^h_p\left(p\right) &= \frac{1}{2\alpha-1}\left(\frac{1-\alpha}{v\left(0\right)-p}-\frac{\alpha}{v\left(1\right)-p}\right), \\ \pi^l_p\left(p\right) &= \frac{1}{2\alpha-1}\left(\frac{1-\alpha}{v\left(1\right)-p}-\frac{\alpha}{v\left(0\right)-p}\right), \end{aligned}$$

whenever

$$\alpha\log\left(\frac{v\left(0\right)}{c}\right)>\left(1-\alpha\right)\log\left(\frac{v\left(1\right)}{c}\right).$$

By definition (in the interior of the relevant supports), these derivatives must be negative.

In any equilibrium, we have always $\pi^h(p) > 0$, and $\pi_p^l(p) < 0$. To have a simple equilibrium, where

$$p \in \text{ supp}\pi^h \cap \text{ supp}\pi^l \Rightarrow [0, p] \subset \text{supp}\pi^h \cap \text{ supp}\pi^l, \tag{6}$$

we should have $\pi_p^h(p) < 0$ for $p < p'$, where $p' = \left\{p : \pi^l(p) = 0\right\}$. This is the case for c high enough. For low c, equilibrium must have a different structure.

We compute the second derivatives of the entry intensities as follows:

$$\begin{aligned}
\pi_{pp}^h(p) &= \frac{1}{2\alpha - 1}\left(\frac{1-\alpha}{(v(0) - p)^2} - \frac{\alpha}{(v(1) - p)^2}\right), \\
\pi_{pp}^l(p) &= \frac{1}{2\alpha - 1}\left(\frac{1-\alpha}{(v(1) - p)^2} - \frac{\alpha}{(v(0) - p)^2}\right).
\end{aligned}$$

We see here that the following implication holds:

$$\pi_p^h(p) \geq 0 \Longrightarrow \pi_{pp}^h(p) > 0,$$

which implies that the only possible equilibrium that does not satisfy condition (6) is the following: $\text{supp}\pi^l(\cdot) = \left[0, p^l\right]$ and $\text{supp}\pi^l(\cdot) = \left[p^l, p^h\right]$ for some $p^h > p^l$. Hence the entry distributions in the first-price auction with many bidders look qualitatively similar to the two-bidder case. We summarize our findings in the following lemma.

Lemma 4 *In the entry game with first-price auction, we have either i)* $0 \in$ *supp* $\pi^s(\cdot)$ *for* $s \in \{h, l\}$ *or ii) supp* $\pi^l(\cdot) = \left[0, p^l\right]$ *and supp* $\pi^h(\cdot) = \left[p^l, p^h\right]$ *for some* $0 < p^l < p^h < v(1)$.

4.2.2 Highest possible price

The highest possible price, $\overline{p}$, can be easily solved by considering the highest bidding high type bidder. He is certain to get the good, so upon getting it his belief is unchanged. On the other hand, free entry means that his value-price margin must be c. So, we have

$$q^h(v(1) - \overline{p}) + \left(1 - q^h\right)(v(0) - \overline{p}) = c$$

or

$$\begin{aligned}\overline{p} &= q^h v(1) + \left(1-q^h\right) v(0) - c \\ &= q^h \Delta v + v(0) - c,\end{aligned}$$

where $\Delta v := v(1) - v(0)$.

4.2.3 Price range above low type

Take $p \in \text{supp}\pi^h(\cdot) \cap \left[supp\pi^l(\cdot)\right]^C$, where θ^h enter but θ^l do not enter. Then by Lemma 4, we have $\pi^l(p) = 0$, and $\pi^h(p) > 0$. Any bidder of type θ^l that submits bid p makes an expected loss:

$$q^l e^{-\alpha\pi^h(p)}\left(v(1) - p\right) + \left(1-q^l\right) e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right) < c$$

and a bidder of type θ^h breaks even in expectation:

$$q^h e^{-\alpha\pi^h(p)}\left(v(1) - p\right) + \left(1-q^h\right) e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right) = c.$$

For this to hold, we must have

$$e^{-\alpha\pi^h(p)}\left(v(1) - p\right) > c > e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right).$$

It is clear that for $p < \overline{p}$, the indifference equation for high type is solved by some decreasing function $\pi^h(p)$.

If $\pi^h(\cdot)$ is constructed to maintain indifference for the bidders with θ^h, then at some p, low type wants to enter. This can always be guaranteed to happen if the optimal entry rate $\widehat{\pi}^l$ in the planners problem is strictly positive. There is some $p' < \overline{p}$ such that

$$q^l e^{-\alpha\pi^h(p)}\left(v(1) - p\right) + \left(1-q^l\right) e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right) > (<)\, c$$

for $p < (>)\, p'$. We find p' by requiring

$$\begin{aligned}& q^l e^{-\alpha\pi^h(p)}\left(v(1) - p\right) + \left(1-q^l\right) e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right) \\ =\; & q^h e^{-\alpha\pi^h(p)}\left(v(1) - p\right) + \left(1-q^h\right) e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right)\end{aligned}$$

and this is of course satisfied when

$$e^{-\alpha\pi^h(p)}\left(v(1) - p\right) = e^{-(1-\alpha)\pi^h(p)}\left(v(0) - p\right) = c.$$

Clearly there is a unique p' that solves this, and $p' < \overline{p}$. So, the range of prices where only high type enters, is $[p', \overline{p}]$.

If c is high enough, we have a simple equilibrium where θ^l submit bids within $[0, p']$ and θ^h submit bids within $[0, \overline{p}]$. Notice that since $\pi^h(0)$ and $\pi^l(0)$ given in (4) and (5) are equal to the efficient entry rates, this equilibrium is efficient, and hence maximizes expected revenue to the seller across all possible mechanisms.

4.2.4 Price range where only low type is active

Suppose that only θ^l submits bids in some $[p^-, p^+]$ for some $0 \leq p^- < p^+$. Let $\overline{\pi}^h := \pi^h(p^+)$ denote the constant entry intensity above $p \in [p^-, p^+]$ for θ^h. Since type θ^h does not want to enter and low type is indifferent, so

$$\begin{aligned} q^h e^{-\alpha\overline{\pi}^h - (1-\alpha)\pi^l(p)} (v(1) - p) + \left(1 - q^h\right) e^{-(1-\alpha)\overline{\pi}^h - \alpha\pi^l(p)} (v(0) - p) &< c, \\ q^l e^{-\alpha\overline{\pi}^h - (1-\alpha)\pi^l(p)} (v(1) - p) + \left(1 - q^l\right) e^{-(1-\alpha)\overline{\pi}^h - \alpha\pi^l(p)} (v(0) - p) &= c\,. \end{aligned}$$

For this to hold, we must have

$$e^{-(1-\alpha)\overline{\pi}^h - \alpha\pi^l(p)} (v(0) - p) > c > e^{-\alpha\overline{\pi}^h - (1-\alpha)\pi^l(p)} (v(1) - p)\,.$$

We can show that for some parameters, there is an equilibrium where θ^l bids within $[0, p'']$ and θ^h within $[0, p'] \cup [p'', \overline{p}]$, where $p' < p''$. Notice that this equilibrium is also efficient.

For other parameters, we can have an equilibrium, where θ^l bids within $\left[0, p^l\right]$ and θ^h within $\left[p^l, \overline{p}\right]$. Since the price support of high type does not extend to zero, this equilibrium is not efficient.

4.3 Revenue comparisons

We have shown that for some cases where both types of bidders enter with positive intensity, zero bids are in the support of the bid distribution for both bidder types. Hence the first-price auction results in socially optimal entry in this case. Since the bidders' expected surplus is zero by construction, expected revenue must equal expected social surplus. This implies that FPA gives the highest possible revenue to the seller subject to individual rationality by the bidders.

We also showed that all equilibria of the SPA involve distorted entry profiles. Since the bidders still make a zero expected profit, this implies that the expected revenue in any symmetric equilibrium of the SPA falls below the expected revenue in FPA.

For small $c > 0$, we have found a numerical example where a symmetric equilibrium of the SPA dominates the FPA in terms of expected revenue. Hence the clean revenue ranking of the two-bidder game no longer holds. We can show, however, that FPA dominates for high enough entry costs.

5 Further remarks

5.1 Entry cost at ex ante stage

In this subsection, we consider equilibria in a version of the model where entry decision is made before learning the signals. This corresponds to the case where inspecting the good for sale is costly. We maintain the assumption that the number of actual bidders that have paid the cost c is not disclosed prior to the bidding stage. This is the only significant modeling difference in comparison to Levin & Smith (1994). The example demonstrate that the randomness in the number of participants allows the bidders in a second-price auction to get higher profit than in the first price auction.

We assume here that there are only two potential entrants. Consider the following special case of the binary model:

$$\alpha^h = \Pr\left\{\theta_i = \theta^h | \omega = 1\right\},$$

and

$$\alpha^l = \Pr\left\{\theta_i = \theta^l | \omega = 0\right\} = 1.$$

In this case, the posteriors are

$$q^h = 1,\ q^l = \frac{q\left(1-\alpha^h\right)}{1-q\left(1-\alpha^h\right)}.$$

The characterization of socially optimal entry using symmetric strategies is even easier than before. Simply compute the symmetric entry probability

π to get

$$\begin{aligned} W(\pi) \quad : \quad &= \max_{\pi} q\left[\left(1-(1-\pi)^2\right)v(1)-2\pi c\right] \\ &+ (1-q)\left[\left(1-(1-\pi)^2\right)v(0)-2\pi c\right]. \end{aligned}$$

Hence the optimal entry rate is given by:

$$\pi^* = \frac{qv(1)+(1-q)v(0)-c}{qv(1)+(1-q)v(0)} := \frac{\overline{v}-c}{\overline{v}}.$$

Consider a bidding equilibrium in a first price auction that is held under the assumption that entry has taken place at rate π^*. Since entry is independent of the true state, both types of bidders believe to be bidding alone with probability $(1-\pi^*)$. With probability π^*, there is a competing bidder. A bidder of type θ^h believes that her opponent is of type θ^h with probability α^h. A bidder of type θ^l believes that her opponent is of type θ^h with probability $q^l\alpha^h$. Hence for θ^h, the three possible events {no competitor, competitor of type θ^h, competitor of type θ^l} have probabilities $\left(1-\pi^*, \alpha^h\pi^*, \left(1-\alpha^h\right)\pi^*\right)$, and for θ^l, the corresponding probabilities are $\left(1-\pi^*, \alpha^h q^l\pi^*, \left(1-\alpha^h q^l\right)\pi^*\right)$. Finally, let $\widehat{q}^l$ denote the posterior probability that a bidder of type θ^l assigns on $\{\omega=1\}$ if she is told that there is no competitor of type θ^h, i.e.

$$\widehat{q}^l := \frac{q^l-\alpha^h q^l\pi^*}{1-\alpha^h q^l\pi^*}.$$

Again, it is easy to see that all possible equilibria involve mixed strategies for both types of players. Assume then first that the supports in the distributions are intervals without overlap such that θ^l bids on $[0,p^l]$ and θ^h bids on $[p^l,p^h]$. Furthermore let

$$v^l := q^l v(1) + \left(1-q^l\right)v(0),$$

and

$$\widehat{v}^l := \widehat{q}^l v(1) + \left(1-\widehat{q}^l\right)v(0).$$

Then we have from the optimality conditions of the two types of bidders:

$$(1-\pi^*)v^l = \left(1-\alpha^h q^l\pi^*\right)\left(\widehat{v}^l-p^l\right),$$

$$\left(1-\pi^*\right) v\left(1\right) \leq \left(1-\alpha^h\pi^*\right)\left(v\left(1\right)-p^l\right) = v\left(1\right)-p^h.$$

If we have $\alpha^h \to 1$, we see from the inequality that p^l must converge to zero. Also, $q^l \to 0$ and $\widehat{q}^l \to 0$ and therefore the first equation becomes

$$\left(1-\pi^*\right) v\left(0\right) = v\left(0\right).$$

Since π^* does not depend on α^h, we get a contradiction. Hence we conclude that for $\alpha^h \to 1$, ordered supports are not possible. By arguments similar to the previous sections, we also conclude that when α^h is large enough, then 0 is in the support of both bid distributions.

In this case, the bidding strategy $\left(b^h, b^l\right) = (0,0)$ is an optimal ex ante strategy for a player considering entry. The expected payoff from this strategy is the value of the object in each state of the world if the other bidder does not enter, i.e.

$$\left(1-\pi^*\right)\left(qv\left(1\right)+\left(1-q\right)v\left(0\right)\right),$$

and the cost is c. Hence by definition of π^*, the player is indifferent between entering and not, and we have an overall equilibrium in the game with symmetric entry rate π^* and zero payoffs to the bidders.

A full analysis of this model (including the many bidders case) is left for future research.

5.2 More signals

A trivial extension to a pure strategy equilibrium in a model with a continuum of signals is possible immediately for our model. Consider a continuum of signals with only two possible likelihood ratios for the states of the world. Lumping signals with the same likelihood ratios into two aggregate signals yields a model that is equivalent to the current discrete-signal case. The mixed strategy equilibria of the current paper can then be translated into a pure strategy equilibrium of the continuous model. As long as the signals satisfy monotone likelihood ratio property, there is good reason to hope that the results would hold for nearby signal structures (in the sense that conditional signal distributions are close to the continuous version of the current model in the weak topology).

More generally, the case with more signals (e.g. a continuum) seems relatively easily handled in the case of a large number of bidders. The planner can concentrate on the two signals with the highest and the lowest likelihood ratios for the states. By mixing these appropriately, any feasible entry profile can be generated. Equilibria in first-price and second-price auctions can then be constructed where only bidders with these signals enter.

5.3 More objects

We are currently working on an extension to the case with k objects for sale. A comparison of discriminatory auctions and $k+1^{st}$ price auctions is then possible. For payoff calculations in the discriminatory auction, each bidder trades off the probability of having one of the k highest bids to the cost of the bid. Hence the equations for bidder indifference are similar to the ones in the first-price auction of Section 4. For the $k+1^{st}$ price auction, the same issues on non-unique equilibrium (pure) bids for θ^l and randomized bids for θ^h persist.

This extension also raises the interesting issue of information aggregation for large k. It seems clear that full information aggregation in the sense of Pesendorfer & Swinkels (1997) or Kremer (2002) will not be possible. As long as $c > 0$, numbers of entrants remain random, and prices in $k+1^{st}$ price auctions will not converge to the true value of the object. Determining the limit distribution of the realized price when $c \to 0$ is also an interesting question for future research.

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