HISTORY-DEPENDENT RECIPROCITY IN ALTERNATING OFFER BARGAINING *

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This paper studies alternating-offer bargaining with players who have history-dependent reciprocity preferences. To allow for reciprocal motivation, the existing history-dependent models are modified by reversing the way aspirations depend on previous offers. The model exhibits a unique equilibrium where an agreement is reached immediately. As the players' discount factors approach unity, players share the pie according to the golden division: the responder's share of the whole pie coincides with the ratio of the proposer's and the responder's shares. Thus, there is a first-mover disadvantage. (JEL: C72, Z13)

1. Introduction

A large body of evidence suggests that reciprocity is an important determinant of interactive human behavior. By reciprocity it is meant that a player prefers behaving kindly towards another player who behaves kindly towards him. On the other hand, unkind behavior is preferred if the other is also unkind. Reciprocal motivations have been shown to be particularly important in bargaining and negotiation contexts (Güth *et al.* 1982; Roth 1995; Camerer 2003).

By now, there are formal economic models which incorporate reciprocity motivations into interactive decision making (Rabin 1993; Dufwenberg and Kirchsteiger 2004; Falk and Fischbacher 2006). In these models reciprocal reac-

tions depend on the perceived kindness of other players' actions which depend on perceived/believed intentions. Thus by definition, reciprocity models generally fall into the category of psychological game theory (Geanokoplos, Pearce, and Stachetti 1989). There is empirical support for this explicit dependence of payoffs on beliefs.[1] Yet, this feature of the reciprocity models is a complication which has drawbacks. First, it requires leaping away from the well-understood traditional game-theoretic framework where payoffs depend solely on the outcomes of the game. Second, in many occasions, belief-based reciprocity generates multiplicity of equilibria, some where all players behave kindly towards each other and others where they behave unkindly, for instance.

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[1] *See for instance Falk et al. (2008).*

Multiplicity and ambiguity is perhaps descriptive of reality. Yet, an important objective of game theoretic research has been to unambiguously state a unique prescription for rational play.[2] In bargaining, the alternating offer protocol (Stahl 1972; Rubinstein 1982) was the key idealization which led to the identification of a unique rational bargaining outcome when players have standard non-interdependent and stationary preferences. Yet, making prescriptions of rational *reciprocally motivated* behavior in such rich extensive form games as the alternating offer bargaining has turned out difficult and not fully transparent. The psychological equilibria depend heavily on the beliefs about counterfactuals and, in complicated game forms; there is an abundance of potential off-path beliefs. Thus, for understanding hypothetical rational play as opposed to predicting actual play, it may be helpful to resort to more idealized models of reciprocity. In his seminal contribution, Rubinstein is primarily interested in the rationality prescription; he is neither asking the positive question of how the pie will be shared nor the normative question of what is the just sharing of the pie (Rubinstein, 1982, p. 97).

There is nothing irrational about being reciprocally motivated.[3] Thus the interest in the reciprocally motivated rational bargaining outcome is a perfectly valid one. Wishing to contribute to our understanding of the rational reciprocally motivated bargaining in the infinite alternating offer environment, this paper proposes a somewhat stylized model of reciprocity. Our model, where the kindness of the opponent depends only on the history but neither on beliefs about off-path nor about future behavior, surely ignores some plausible aspects of reciprocally motivated behavior. Yet, it allows for a transparent analysis and an unambiguous prescription for history-dependent reciprocity alone.

More specifically in the model, players' reciprocity preferences are embodied in aspirations which depend inversely on the history of the kindness of the opponent's proposals. Each player is assumed to prefer rejecting any offer which would give her less than she aspires. This modelling approach is inspired by Fershtman and Seidmann (1993) and Li (2007) which also study the influence of history-dependent preferences on alternating-offer bargaining. These models are not intended to capture *reciprocity motivations* but to consider players who do not want to accept offers which are worse than previous offers. Therefore an unkind proposal induces a lower aspiration than a kind proposal and thus unkindness is rewarded and kindness is punished. Our model fine-tunes the history-dependent aspirations to account for reciprocal preference-responses. This is achieved by reversing the dynamics of the aspiration levels: unkind proposals lead to higher aspirations (if rejected) whereas kinder proposals increase aspiration less.

Focusing on history-dependence alone allows for a unique prescription for reciprocally motivated alternating offer bargain. When players become infinitely patient, the unique subgame-perfect equilibrium of the game predicts that the cake will be shared according to the golden division. There is a first-mover disadvantage: the responder's share of the whole pie coincides with the ratio of the proposer's and the responder's shares. Thus, the responder immediately receives $1/\varphi$ of the cake where φ is the golden number.[4]

We only consider the infinite horizon model in this paper, but simple one or two period models help to see the limits of the model. With one or two rounds of proposals, history-dependent reciprocity has little bite. This emphasizes the limited scope of the approach in explaining empirical regularities in the ultimatum game data, for instance, which the belief-dependent models provide an interesting way to rationalize. In the ultimatum game, clearly it is optimal to make the smallest proposal which the responder accepts. This offer coincides with the responder's aspiration in the present model. In the ultimatum game, the player receiving a proposal never faced a proposal before and thus his aspiration is entirely exogenous to the model. If the responder's exogenously given aspiration is zero,

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[2] *Harsanyi and Selten (1988), for instance, very explicitly pursue this agenda.*

[3] *Smith (2008), for instance, argues that tractability has motivated, not only the neglect of reciprocity, but of many other relevant aspects of human decision making in economic modeling. Modeling reciprocity does not require violations of completeness, reflexivity, or transitivity – merely an extension of the outcome space.*

[4] $\varphi = (1 + \sqrt{5})/2 \approx 1.6180339887...$

the optimal proposal leaves the responder with nothing at all, a solution clearly at odds with observed empirical regularities. Then again assuming individual heterogeneity in initial aspirations allows one to fit any data perfectly by suitably meddling with the initial aspiration densities. Also theoretically, this short horizon model provides little new insight – the solution coincides with those of the models of Stahl (1972)[5], Fershtman and Seidmann (1993), and Li (2007). This simple example should make clear that the nature of the exercise of the paper is not a descriptive one of predicting data but more like a thought experiment isolating a hypothetical factor of interest which has not been captured before due to having to deal with complicated models.

As suggested above, our model is closely related to Li (2007), Fershtman and Seidmann (1993) as well as Compte and Jehiel (2004) which assume that bargaining history endogenously influences the environment in a way which has been traditionally assumed exogenous to the bargaining interaction.[6] These papers point out that such endogeneities may lead to gradual concessions and delays in bargaining due to having to hold back generous instantaneous offers which might lead to a disproportionate improvement of the opponent's bargaining position. Compte and Jehiel (2004) suppose that a generous offer improves the best outside option of the player receiving such an offer. Fershtman and Seidmann (1993) analyze the combined effect of a deadline and a preference for rejecting all offers below the highest offer so far. This combination is shown to lead to delays. Yet, without a deadline, there is no delay – the standard stationary solution proposed by Rubinstein (1982) is an equilibrium in that case.

Li (2007) assumes that aspirations are tied to payoffs: a player prefers rejecting all offers which would generate a lower payoff than the player would have achieved by accepting an earlier offer. Li concludes that there may be delay even without a deadline. The present paper illustrates that endogenous aspirations alone do not imply delay, rather the positive association of aspirations and previous proposals does. If the aspirations are inversely related to previous offers, as reciprocal motivations call for, then the agreement is immediate and the first-mover is disadvantaged when players are sufficiently patient. This result holds when there is no deadline whether the aspirations are tied directly to offers (Fershtman and Seidmann 1993) or to discounted payoffs (Li 2007).

The paper is organized as follows. In section 2, the model is presented. Section 3 solves and analyses the equilibrium in the case of the reciprocal Li-model. The fairly analogous analysis of the reciprocal FS-model is relegated to the appendix in Section 5.2. Section 4 concludes.

2. The model

Two impatient players bargain over a pie of size 1 according to the infinite horizon alternating-offers procedure. The future is discounted at a common discount factor δ. An agreement is a vector $x = (x_1, x_2)$ in which x_i is player i's share of the pie. The set of all feasible agreements is $X = \{x \in [0,1] : x_1 + x_2 = 1\}$. An agreement can be reached in period $t \in T = \{0, 1, 2, ...\}$. An impasse (perpetual disagreement) is denoted by D.

Bargainers may have aspirations. Each player is reluctant to strike a deal where her share of the pie is smaller than her aspired share of the pie. The aspiration may evolve over time. The aspiration of player i at time t is denoted by z_i^t and the vector of aspirations at time t is denoted by z^t. An outcome in the outcome space is $o \in O = \{X \times Z \times T\} \cup \{D\}$. Strategy and history are defined as usual.

Formally, the payoffs can be written as follows.[7] The payoff $U_i : O \longrightarrow [0,1] \cup \{-\varepsilon\}$ of player i is $U_i(o\,;\delta) = \delta^t x_i$ if $o = (x, z, t)$ is such that $x_i \geq z_i^t$; perpetual disagreement gives payoff zero, $U_i(o\,;\delta) = 0$ if $o = D$ or if $\sum_i z_i^t > 1$; receiving a share below one's aspiration yields a small negative payoff: $U_i(o\,;\delta) = -\varepsilon$ if $x_i < z_i^t$, where ε is a small positive constant.[8]

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[5] *If one assumes a zero initial aspiration in the present model.*

[6] *See also Compte and Jehiel (2003).*

[7] *See also Li (2007).*

[8] *Alternatively, one could consider a lexicographic preference for perpetual impasse over accepting an offer below one's aspiration.*
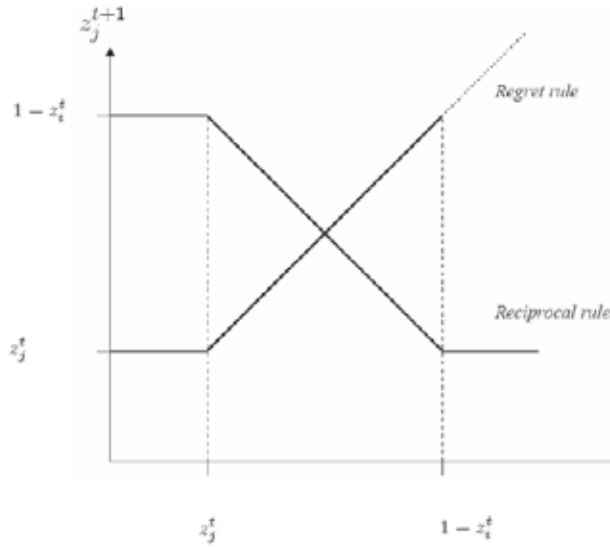
Figure 1. Regret rule and reciprocal rule based on Fershtman and Seidmann (1993)

Let us denote the proposal in period t by c^t. A proposal $c^t \in [0, 1]$ at round t gives the responder the corresponding share of the pie if the responder accepts. The proposer receives the residual, $1 - c^t$, if the offer is accepted. If the offer is rejected, play proceeds to the follow-up round with the responder of the preceding round proposing in his turn.

Let us now turn to the history-dependent dynamics of the aspirations, z^t. The transition rule of the aspiration from round t to round $t + 1$ is a mapping from the proposal, c^t, and the aspirations, $z^t = (z_1^t, z_2^t)$, in round t, to the aspiration in round $t + 1$

$$z_i^{t+1}(c^t, z^t; \delta) : [0, 1]^2 \longrightarrow [\underline{b}(z_i^t; \delta), \overline{b}(z_j^t; \delta)]$$

with initial aspirations z_i^0, $z_j^0 \in [0, 1]$. Here $\underline{b}(z_i; \delta)$ and $\overline{b}(z_j; \delta)$ are the lower and upper bound of the aspiration at round $t + 1$, respectively, and they may depend on the current aspirations. This general transition rule trivially subsumes Rubinstein's (1982) model with discount factors as a special case with $z_1^0 = z_2^0 = 0$ and $z_i^{t+1}(c^t, z^t; \delta) = 0$ if $z^t = (0, 0)$ for any c^t. It subsumes the transition rule of Fershtman and Seidmann (1993) as a special case: if j is the

responder at t, his aspiration at round $t + 1$ is $z_j^{t+1}(c^t, z^t) = \max\{c^t, z_j^t\}$ and round t proposer's aspiration at $t + 1$ is $z_i^{t+1}(c^t, z^t) = z_i^t$ and thus the lower bound of the aspiration is always the current aspiration, $\underline{b}(z_k^t) = z_k^t$, for $k = 1, 2$. This models players who refuse to accept lower offered shares than the highest previously offered share. Finally, the transition rule of Li (2007) satisfies $z_i^{t+1}(c^t, z^t; \delta) = \max\{c^t/\delta, z_i^t/\delta\}$ for the responder at t and $z_i^{t+1}(c^t, z^t; \delta) = z_i^t/\delta$ for the proposer at t and thus the lower bound of the next round aspiration is the next-round value of the current aspiration, $\underline{b}(z_i^t; \delta) = z_i^t/\delta$. This models a player who refuses to accept any proposal which gives lower payoff than the highest payoff proposal so far had the player accepted when it was made.

Let us now proceed to motivate the aspiration transition rules in the current paper. With existing models of reciprocity, analyzing reciprocity in the alternating offers context has proven insurmountable due to the dependence of preferences not only on the history but also on the belief about the other's strategy and on the belief of the other about one's own strategy and so forth. In order to study the implications of reciprocity in this context in a simple and transpar-
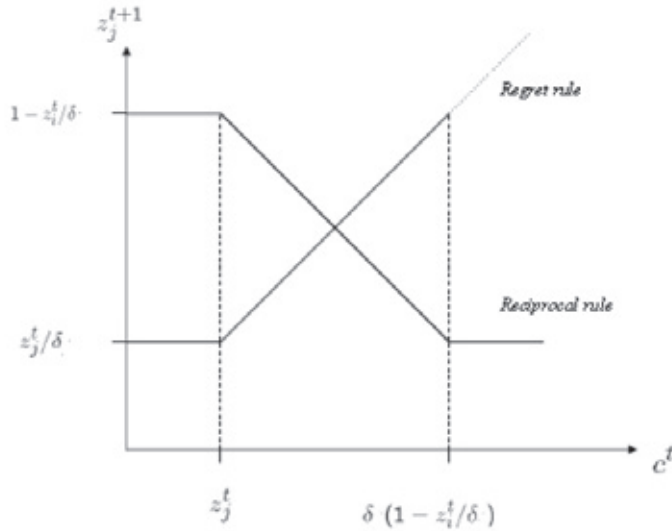
Figure 2. Regret rule and reciprocal rule based on Li (2007)

ent manner, I consider a merely history-dependent approach inspired by existing models where players have history-dependent aspirations.

In the history-dependent aspiration models, the so called *regret models* (Fershtman and Seidmann 1993; Li 2007), formally stated above, a generous history of offers by the opponent leads to the player having a higher aspiration whereas the opponent's stingy offers lead to a lower aspiration. This is diametrical to the core feature of reciprocal preference, the willingness to reward kindness and punish unkindness. With reciprocal history-dependence, generous offers should lead to a lower aspiration whereas stingy offers should lead to a higher aspiration.

The simplest way to modify the existing models to accommodate for reciprocity is to reverse the linear dependence of the aspiration at $t+1$ on the proposal at t given the bounds set by the aspirations at t. This reaches the following transition rules in the model where aspirations are directly tied to proposals (see Figure 1):

$$z_j^{t+1} = \begin{cases} 1 - z_i^t & \text{if } c_t < z_j^t \\ z_j^t + 1 - z_i^t - c^t & \text{if } c_t \in [z_j^t, 1 - z_i^t] \\ z_j^t & \text{if } c_t > 1 - z_i^t \end{cases}$$

for the responder and $z_i^{t+1}(c^t, z^t) = z_i^t$ for the proposer at t. Let us call this *the reciprocal FS-model*. In the initial or regret-based FS-model, a responder's aspiration remains unchanged after rejection if the offer is below or equal to his aspiration. On the other hand, a player's aspiration coincides with the residual of the proposer's aspiration if the proposer offers the whole pie less of her aspiration and is rejected. In the reciprocal rule these associations are reversed. Moreover, the transition rule in the reciprocal FS-model linearly connects these endpoints just as it does in the regret-based FS-model. Thus, if the proposal c^t to the responder j satisfies $c^t \in (z_j^t, 1 - z_i^t)$, the period $t+1$ aspiration of the responder is the current one plus how much more than his aspiration the proposer proposes to himself. This yields a transition rule where a more generous proposal leads to a lower aspiration if rejected so that generously proposing the residual of one's own aspiration yields the lower bound and stingily proposing the opponent's current aspiration yields the upper bound.

Imposing the upper bound rules out strategic threats of impasse in the follow-up round by the current proposer. If, in a given round, there was an offer that led to incompatible positions in the

next round and some other which did not, the one leading to an impasse would have to be the most unkind one due to our monotonicity assumption. On the other hand, the responder would have to accept this offer due to the threat of an impasse. The transition of the aspiration in the present model avoids such trivialities by assuming that aspirations are compatible whatever the offer.

Alternatively, tying aspirations to discounted payoffs, as in the Li-model, yields the following formulation (see Figure 2):

$$z_j^{t+1} = \begin{cases} 1 - z_i^t/\delta & \text{if } c_t < z_j^t \\ z_j^t/\delta + 1 - z_i^t/\delta - c^t/\delta \\ & \text{if } c_t \in [z_j^t, \delta(1 - z_i^t/\delta)] \\ z_j^t/\delta & \text{if } c_t > \delta(1 - z_i^t/\delta) \end{cases}$$

for the responder and $z_i^{t+1} = min\{1, z_i/\delta\}$ for the proposer at t. This will be called the *reciprocal Li-Model* in the sequel.

In the initial or regret-based Li-model the transition rule is as follows. If the round t offer is below or equal to the responder's round t aspiration, this latter is multiplied by the inverse of the discount factor to yield round $t + 1$ aspiration, which in fact is nothing but the round $t + 1$ value of the round t aspiration. On the other hand, the responder's aspiration coincides with the residual of the proposer's next round aspiration if the proposer offers the round t value of the residual of her round $t + 1$ aspiration and is rejected. Again, the reciprocal Li-model reverses these associations and linearly connects these endpoints just as it does in the FS-model thus yielding a transition rule where a more generous proposal leads to a lower aspiration if rejected. Proposing at round t the discounted residual of one's own round $t + 1$ aspiration yields the lower bound and proposing the responder's current aspiration yields the upper bound. If the proposal c^t to the responder j satisfies $c^t \in (z_j^t, \delta(1 - z_i^t/\delta))$, the period $t + 1$ aspiration of the responder is the round $t + 1$ value of the current one plus how much more than his round $t + 1$ aspiration the proposer proposes to himself in period $t + 1$ value. As in the reciprocal FS-model, the upper bound is imposed to rule out trivialities.[9]

3. Solution

Before solving the unique equilibrium under the reciprocal Li-model, let us first establish that, both in the reciprocal FS-model and in the reciprocal Li-model, the sum of aspirations increases if an equilibrium proposal is rejected. For the reciprocal Li-model, this follows by construction since aspirations in the follow-up round are at least z_i/δ for $i = 1, 2$. In the reciprocal FS-model this feature holds in equilibrium as will be first verbally argued and then formally proven in Lemma 1. The remaining parts of the proof are analogous for the two models and thus formal proofs under the FS-model are relegated to Appendix B.

Crucial is to notice that, with reciprocal aspirations, the proposer faces an essential trade-off between the share that the proposer receives, if the offer is accepted, and the worsened bargaining position due to the increase in the opponent's aspiration[10], if the offer is rejected. Proposing the share which would not alter the responder's future aspiration will be accepted for sure. Namely, this most generous feasible offer results in the highest possible payoff for the responder at any history to come. The most generous feasible offer is thus hardly the best for the proposer. Thus, only offers which improve the responder's bargaining position, if rejected, will ever be proposed.

The fact that the offer which does not alter aspirations is never made implies that aspirations keep on increasing if optimal offers are rejected. Thus, repeated rejections would lead aspirations to approach just-compatibility.[11] Eventually, there would thus be a round where, if the responder's aspired share is proposed, the responder accepts knowing that in the follow-up round the current proposer's aspiration is so high that even getting the one-period delayed

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[9] *It is assumed that, if there is an offer which makes aspirations incompatible, then every offer makes aspirations incompatible.*

[10] *In the reciprocal Li-model, the worsened bargaining position is reflected in the relatively greater increase in the responder's aspiration than in the proposer's.*

[11] *The sum of aspirations would approach one. This holds in the reciprocal FS-model.*

residual of it is not preferable to accepting one's own aspiration immediately, $\delta(1 - z_p) < z_r$. At such a round the responder will accept any feasible proposal. The set of aspirations which satisfy this condition will be denoted by I^0.

In the reciprocal FS-model I^0 is characterized by $1 < z_r/\delta + z_p$ and $1 \geq z_r + z_p$ where z_p and z_r are the aspirations of the proposer and the responder, respectively, and $1 \geq z_r + z_p$ rules out impasse. In the reciprocal Li-model the set of aspirations where responder prefers accepting all offers is even larger, $I^0 = \{z | 1 \geq z_r + z_p > \delta\}$. This is due to the fact that, if the optimal offer is rejected, even the proposer's aspiration will be higher and thus the discounted residual will be smaller. In fact the responder when $z \in I^0$ accepts knowing that rejection of any offer will lead to incompatible aspirations and thus to an impasse.

Lemma 1. *If equilibrium proposals were rejected, aspirations would eventually reach I^0.*

Proof. In the reciprocal Li-model, this holds by construction. Thus, consider the reciprocal FS-model and an arbitrary round t where WLOG j is the proposer. In round $t + 1$, j will reject any offer smaller than z_j^t. If j accepts $c^{t+1} < z_j^t$ then her payoff is $-\varepsilon$ which is smaller than 0, the minimum she would obtain if she rejected any offer and always proposed $1 - z_j^\tau$ at $\tau \geq t$. Thus, i will not receive more than $(1 - z_j^t)$ in follow-up round if he rejects the offer of j in round t. Therefore, j will propose at most $\delta(1 - z_j)$ in round t. Thus, $\widehat{c}^t \leq \delta(1 - z_j) < 1 - z_j$ and therefore the sum of aspirations in round $t + 1$ will be strictly greater than the sum of aspirations in t, $z_i^t + z_j^t < z_i^{t+1} + z_j^{t+1}$, if the equilibrium offer is rejected at t.

The optimal proposal is the one which leaves the responder indifferent between accepting and rejecting. The offer is so generous that despite the prospect of an improved bargaining position, the improvement is barely sufficient to cover the cost of delaying the agreement. Therefore, the responder accepts the offer. The responder strictly prefers accepting immediately any offer larger than the optimal offer and rejecting any offer

smaller than the optimal offer. This is a feature shared with the standard solution (Rubinstein 1982). The reciprocal problem differs from the stationary one in that the optimal offer must be more generous in the former problem since the prospect of an improved bargaining position invites the responder to reject a larger number of offers. It is easy to see that the optimal proposal when aspirations are in I^0 is z_r. Let us define $\widehat{c}^0$ as that the optimal proposal given $z \in I^0$.

Let us focus on the reciprocal Li-model for the rest of the section. The solution of the reciprocal FS-model proceeds analogously and it is thus left to the appendix. Let us now consider the characterization of I^1 where, by definition, rejecting the optimal offer yields aspirations to lie in I^0. That is, the aspirations are such that the optimal offer, if rejected, leads to the sum of follow-up-round aspirations being in the interval I^0 or formally

$$1 \geq (z_r/\delta + 1 - z_p/\delta - \widehat{c}^1/\delta) + z_p/\delta > \delta,$$

where z_p and z_r are the aspirations in I^1 and $\widehat{c}^1$ is the optimal proposal at that round. In the expression, the first term, in the brackets, and the second term, z_p/δ, describe the transition to aspirations z_p' and z_r' in the follow-up round, I^0, respectively (notice that, since the roles are reversed, so are the subindices when moving across periods).

Since the optimal proposal when $z \in I^0$ is $\widehat{c}^0$ and this proposal is accepted, the share that the proposer of the follow-up round would receive at that round is $1 - \widehat{c}^0$. At the round before, this player is indifferent between accepting share $\delta(1 - \widehat{c}^0)$ and rejecting it. Moreover the proposer when $z \in I^1$ strictly prefers $1 - \delta(1 - \widehat{c}^0)$ at that round to $\widehat{c}^0$ in the follow-up round. These two facts imply that proposing $\delta(1 - \widehat{c}^0)$ is strictly preferred to proposing a smaller share than $\delta(1 - \widehat{c}^0)$ when $z \in I^1$. Naturally, the proposer when $z \in I^1$ prefers proposing the smallest share among those that are accepted. Thus, $\widehat{c}^1 = \delta(1 - \widehat{c}^0)$. Plugging in the expression for yields the expression for $\widehat{c}^0$ yields the expression for $\widehat{c}^1 = \delta(1 - z_p/\delta)$.

Generally, let I^k be defined as an interval such that if $z \in I^k$ then k rejections of the optimal offers in the following k rounds would lead as-

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In the reciprocal Li-model, the aspirations would eventually become incompatible due to increases in aspirations of at least $1/\delta$ -magnitude.

pirations to lie in I^0. Let us define the optimal proposal if $z \in I^k$ as $\widehat{c}^k$.

Solving iteratively for $k > 1$ yields

(1)

$$\widehat{c}^k(z) = -\frac{1}{a_{k+1}}(a_k z_p - a_{k-1} z_r$$

$$+ \sum_{l=1}^{k} a_l(-\delta)^{k-l+1})$$

and

(2)

$$I^k = [a_{k+1}\delta^{k+1} - \sum_{l=0}^{k-2} a_{k-(l+1)}\delta^{k-l},$$

$$a_k\delta^k - \sum_{l=0}^{k-3} a_{k-(l+2)}\delta^{k-(l+1)}),$$

where $\{a_n\}_{n=1}^{\infty}$ is the Fibonacci sequence defined iteratively as, $a_1 = 1$, $a_2 = 1$, and for $k > 2$, $a_k = a_{k-1} + a_{k-2}$.

Proposition 2. *Let* $k, \tau \in \mathcal{N}$. *The proposal in period* τ *is* $\widehat{c}^k(z^\tau)$ *if and only if* $z^\tau \in I^k$. *In particular, the proposal in period* 0 *is* $\widehat{c}^k(z^0)$ *if and only if* $z^0 \in I^k$. *Each offer is accepted immediately.*

Proof. The proof for $k = 0, 1$ was given in the main text. Take an arbitrary k and suppose now that the optimal proposal is $\widehat{c}^k(z)$ if and only if $z \in I^k$. Let us then show that $\widehat{c}^{k+1}(z)$ if and only if $z \in I^{k+1}$.

Since the optimal proposal when $z \in I^k$ is $\widehat{c}^k$ and this proposal is accepted, the share that the proposer when $z \in I^k$ receives is $1 - \widehat{c}^k$. Thus, the responder when $z \in I^{k+1}$ is indifferent between accepting share $\delta(1 - \widehat{c}^k)$ and rejecting it. She strictly prefers accepting any larger share and strictly prefers rejecting any smaller share, since a proposal $c < \widehat{c}^{k+1}$ results in smaller optimal proposal $c' < \widehat{c}^k$ and thus a higher payoff for the proposer when $z \in I^k$. Moreover the proposer when $z \in I^{k+1}$ strictly prefers $1 - \delta(1 - \widehat{c}^k)$ at that round to $\widehat{c}^k$ in the follow-up round. These two facts imply that proposing $\delta(1 - \widehat{c}^k)$ is strictly preferred to proposing a smaller share than $\delta(1 - \widehat{c}^k)$ when $z \in I^{k+1}$. Naturally, the proposer when $z \in I^{k+1}$ prefers

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¹³ *See Appendix A on Fibonacci sequence.*

proposing the smallest share among those that are accepted. Thus, $\widehat{c}^{k+1} = \delta(1 - \widehat{c}^k)$. Plugging in the expression for $\widehat{c}^k$ and simplifying yields the desired expression for $\widehat{c}^{k+1}$.

The sum of aspirations must lie in I^k if $\widehat{c}^k$ is rejected. Thus,

$$a_k\delta^{k+1} - \sum_{l=0}^{k-2} a_{k-(l+2)}\delta^{k-l}$$

$$< (z_r/\delta + 1 - z_p/\delta - \widehat{c}^{k+1}/\delta) + z_p/\delta$$

$$\leq a_{k-1}\delta^k - \sum_{l=0}^{k-3} a_{k-(l+3)}\delta^{k-(l+1)}$$

Plugging in the expression for $\widehat{c}^k$ and solving for $z_r + z_p$ delivers the desired expression. It is easy to see that there exists k' depending on δ such that the intervals $\{I^k\}_{k=0}^{k'}$ cover $[0, 1]$ but do not overlap.

Letting players become infinitely patient, $\delta \to 1$, yields the golden sharing. To prove the result, let us first show that when sum of (initial) aspirations is strictly below one and discount factor approaches one, then the number of rounds of rejected equilibrium offers before $z \in I^0$ increases without bound. Then let us show that the optimal proposal $\widehat{c}^k$ approaches the golden share when $z = (0, 0)$ and $k \longrightarrow \infty$.

Lemma 3. *If* $z_i + z_j < 1$, *then for each* $k > 0$ *there is* δ *sufficiently close to one such that for all* $k' \leq k$, $z \notin I^{k'}$.

Proof. It is easy to see that as δ approaches one, the lower bound in each interval I^k, $a_{k+1}\delta^{k+1} - \sum_{l=0}^{k-2} a_{k-(l+1)}\delta^{k-l}$, approaches 1. Thus, the claim.

We are now ready to state the limit result and to give a formal proof.

Proposition 4. *Let* $z_1^0 = 0 = z_2^0$. *As* $\delta \longrightarrow 1$ *in the unique subgame-perfect equilibrium, in the first period , the proposer offers the responder a share*

$$lim_{k \longrightarrow \infty} \widehat{c}^k(z^0) = \frac{1}{\varphi}$$

where φ *is the golden number. The responder accepts the offer.*

Proof. The optimal proposal as a function of number of periods to the end of scope was given by (1). Let us define

$l_{n,k} = \frac{a_{k-n}}{a_k}$ for $n = 1, 2, 3, \ldots$. Thus the optimal proposal reads

$$\widehat{c}^k(z) = l_{2,k}z_r - l_{1,k}z_p + \delta(1 - \sum_{\tau=1}^{k} l_{\tau,k}(-\delta)^{\tau+1}).$$

Let us now define $l_n = \lim_{k\to\infty} l_{n,k}$. By the lemma above, for each $k > 0$ there is delta sufficiently close to one such that for all $k' \leq k$, $z \notin I^{k'}$. Moreover, as $k \to \infty$, the former coefficient, $l_{1,k}$, approaches the inverse of the golden number[12] $1/\varphi$ and the latter coefficient, $l_{2,k}$, approaches $1/\varphi^2$. The sum $-\frac{1}{a_k}(\sum_{l=1}^{k} a_l(-\delta)^{k-l+1})$ also approaches[13] $1/\varphi$.

and therefore, the optimal proposal satisfies

$$\lim_{k\to\infty} \lim_{\delta\to 1} \widehat{c}^k(z) = l_2 z_r - l_1 z_p + l_1$$

and therefore, the first round proposal satisfies

$$\lim_{k\to\infty} \lim_{\delta\to 1} \widehat{c}^k(z^0) = l_2 z_r^0 - l_1 z_p^0 + l_1.$$

In particular, if $z_1^0 = 0 = z_2^0$, then

$$\lim_{k\to\infty} \lim_{\delta\to 1} \widehat{c}^k(z^0) = l_1 = \frac{1}{\varphi} = \Phi$$

which is the reciprocal of the golden number.

This section focused on the derivation of the equilibrium of the reciprocal Li-model. The case of the reciprocal FS-model, derived in the appendix, is analogous. The optimal offers and the conditions on aspiration levels at which the offers are made differ substantially depending on whether the evolution of the Fershtman and Seidmann (1993) model or the Li (2007) model is reversed to accommodate reciprocity. Yet when players are infinitely patient, the offers approach each other. In the limit the pie will be shared according to the golden rule. There is a first-mover disadvantage.

4. Discussion

This paper analyzes reciprocal motivations of rational players in the alternating offer bargaining framework. Unlike in more general approaches to reciprocity (Rabin 1993; Dufwenberg and Kirchsteiger 2004; Falk and Fischbacher 2006), the preferences in our model only depend on the history of rejected proposals, rather than also on the parties' beliefs. Thus, the model abstracts from willingness to currently reward kind future intentions and punish unkind future intentions, which seems descriptive of actual behavior. Perhaps a more important shortcoming of the present model is that it entirely abstracts from the influence of the current proposal on current intrinsic reciprocal preference of the responder – current proposals only influence the future intrinsic preference. Thus the model largely fails to capture the tendency of responders to reject unfair offers in ultimatum bargaining.

Indeed, the model should be taken as a thought-experiment – a first step in trying to analyze reciprocity in infinite alternating offer context. The abstractions avoid the problems of generating multiple equilibria and relying heavily on off-path beliefs which are present if the explicit belief-dependent motivations are modeled within alternating offer bargaining. The abstraction thus allows deriving a unique prescription for a simple form of rational reciprocity-motivated bargaining.

The assumed functional form of the reciprocal dependence of the aspiration on the proposal is of course somewhat ad hoc. As in Fershtman and Seidmann (1993) and Li (2007), a simple form of instantaneous utility where $u(x) = x$ if $x \geq z$ was adopted and the aspirations were set to zero at the start of the bargaining process. A topic for further research would be an attempt to axiomatize the solution itself or to derive utility functions and time-dependencies from more primitive axioms, as is done in Rubinstein and Fishburn (1982). They derive time preferences allowing for both impatience and procrastination from rather standard axioms but assume by means of an axiom that the time neutral outcome is independent of history. Relaxing this latter axiom, among others, might yield a generalization of the preferences studied here (as well as those studied in Li (2007) and in Fershtman and Seidmann (1993)) where an additional axiom puts conditions on how time-neutral outcome, i.e. the aspiration, depends on the history of offers. In general, the offer that has been rejected might impact even the proposer's aspiration. The simple piecewise linear aspiration tran-

sition rule was adopted here as a starting point since it seems like the minimal change to the specification of Li and to that of Fershtman and Seidmann which still allows to account for reciprocal motivations. Whereas a positive association of the previous offer and current aspiration leads to delay (Fershtman and Seidmann 1993; Li 2007) and a first-mover advantage (Li 2007), with a negative association, there is an immediate agreement and a second-mover advantage in the patient limit.

The ordering of turns to propose is exogenous and alternating in the present model. An interesting extension would allow players to pass on the initiative to the opponent.[14] This option might again lead to delay. Indeed, polite suggestions that the other should start are often observed in bargaining contexts with strong fairness cues where reciprocal motivation is likely to matter. Moreover, the one eventually making the first proposal often takes good care of appearing generous and not irritating the other by making too greedy claims of the pie as hinted by the present model.

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[14] *I am grateful to an anonymous referee for making this point.*

Appendix A. Fibonacci series

Let $\{a_1, a_2, a_3, ...\}$ be the Fibonacci sequence. That is $a_1 = 1$, $a_2 = 1$ and for $k > 2$, $a_k = a_{k-1} + a_{k-2}$. Let us define $\frac{a_{k-n}}{a_k} = l_{n,k}$ and $l_n = \lim_{k \to \infty} l_{n,k}$. Notice that since $a_k = a_{k-1} + a_{k-2}$, we have; $1 = l_{1,k} + l_{2,k}$. Or, $l_{2,k} = 1 - l_{1,k}$. Since $a_{k-2} = a_k - a_{k-1}$, we have

$$l_{k,n} = \frac{a_{k-n}}{a_k} = \frac{a_{k-n+2} - a_{k-n+1}}{a_k} = l_{n-2,k} - l_{n-1,k} \ .$$

Thus, $l_n = l_{n-2} - l_{n-1}$.

Lemma 5. $1 + \displaystyle\sum_{\tau=1}^{\infty} l_\tau = 2 + l_1$

Proof. For $n > 3$, replacing $l_n = l_{n-2} - l_{n-1}$ yields

$$1 + \sum_{\tau=1}^{\infty} l_\tau = 1 + l_1 + l_2 + l_1 - l_2 + l_2 - l_3 + l_3 - l_4 + l_4 - l_5 + ... = 2 + l_1.$$

Lemma 6. $1 - \displaystyle\sum_{\tau=1}^{\infty} l_\tau(-1)^{\tau+1} = 1 + \displaystyle\sum_{\tau=1}^{\infty} l_\tau(-1)^\tau = l_1$

Proof. Since $l_n = l_{n-2} - l_{n-1}$, we have

$$1 + \sum_{\tau=1}^{\infty} l_\tau(-1)^\tau$$
$$= 1 - l_1 + l_2 - l_1 + l_2 + l_2 - l_3 - l_3 + l_4 + l_4 - l_5 - l_5 + l_6 + ...$$
$$= 2 - 3l_1 + 2l_2 - 2l_3 + 2l_4 - 2l_5 + 2l_6 + ...$$

where the latter equality follows from $l_{2,k} = 1 - l_{1,k}$. Let us define

$$S = 1 + \sum_{\tau=1}^{\infty} l_\tau(-1)^\tau. \text{ Thus, } S = 2S - l_1 1 + \sum_{\tau=1}^{\infty} l_\tau(-1)^\tau. \text{ Thus, } S = 2S - l_1.$$

implying the claim.

Lemma 7. $1 + \displaystyle\sum_{\tau=1}^{k} l_\tau(-1)^\tau = l_1 + 2l_k(-1)^k - (-1)^k l_{k-1}.$

Proof. Extending the sum and replacing $l_n = l_{n-2} - l_{n-1}$ for $n > 2$ gives

$$
\begin{aligned}
1 + \sum_{\tau=1}^{k} l_\tau(-1)^\tau &= 1 - l_1 + l_2 - l_3 + l_4 - l_5 + l_6 - ... + l_k(-1)^k \\
&= 1 - l_1 + l_2 - l_1 + l_2 + l_2 - l_3 - l_3 + l_4 + ... \\
&\quad ... + (-1)^k (l_{k-2} - l_{k-1}) \\
&= 1 - 2l_1 + 3l_2 - 2l_3 + 2l_4 - \\
&\quad ...(-1)^k 2l_{k-2} - (-1)^k l_{k-1} \\
&= 2 - 3l_1 + 2l_2 - 2l_3 + 2l_4 - \\
&\quad ...(-1)^k 2l_{k-2} - (-1)^k l_{k-1}
\end{aligned}
$$

Let S denote the value of the sum. Thus, $S = 2S - 2l_k(-1)^k + (-1)^k l_{k-1} - l_1$ implying the solution.

Appendix B. Reciprocal FS-model

When the aspirations satisfy

$$(3) \qquad z_r \geq \delta(1 - z_p)$$

where z_p and z_r are the aspirations of the proposer and the responder, respectively. The responder prefers an agreement where he receives z_r to receiving the residual of the aspiration of p in the follow-up round, $\delta(1 - z_p)$. Thus, r prefers accepting any proposal $c \geq z_r$. It is easy to see that the optimal proposal is $\widehat{c}^0 = z_r$ if $z \in I^0$.

Proposing $\delta(1 - \widehat{c}^0)$ in the round before makes the responder at that round indifferent between accepting and rejecting. By standard arguments which also parallel those in the reciprocal Li-model, $\widehat{c}^1 = \delta(1 - \widehat{c}^0)$. This proposal is optimal if and only if rejection of the offer implies that aspirations satisfy (3). Therefore, $z \in I^1$ can be defined by means of two conditions. The complement of (3) holds,

$$\delta \quad > \quad z_j + \delta z_i$$

and, if the optimal offer is rejected, then the aspirations satisfy (3). That is

$$(4) \qquad (1 - \delta(1 - \delta))z_j + \delta z_i \geq \delta^2$$

where the expression follows from plugging in (3) the responder transition rule into z_p, and the optimal proposal into the latter and simplifying.

We can now proceed iteratively to derive, for each $k > 1$ the bounds of each set I^k and the optimal proposal $\widehat{c}^k$ at that round. Let us now consider $k = 2$ defined as the round where rejection of the current optimal offer and the optimal offer in the follow-up round would imply that aspirations will lie in I^0. Proposing $\delta(1 - \widehat{c}^1)$ at such a round makes the responder indifferent between accepting and rejecting. The responder would prefer rejecting any smaller offer and accepting any larger offer since the following round proposer aspiration is decreasing in c and thus the optimal proposal when $z \in I^1$ which decreases in $z_p'(z, c)$ and thus increases in c. The proposer at $k = 2$ strictly prefers $1 - \delta(1 - \widehat{c}^1)$ to $\widehat{c}^1$ at the follow-up round. Thus, p strictly prefers proposing $\delta(1 - \widehat{c}^1)$ to proposing any smaller share. Naturally, among the offers that are accepted by r, $\delta(1 - \widehat{c}^1)$ is the preferred one since the share proposed to the responder is the smallest. Thus, the optimal offer at $k = 2$ is

$$\widehat{c}^2 = \frac{\delta + \delta^2(z_i - z_j)}{1 + \delta^2}$$

On the other hand, $z \in I^2$, if and only if the complement of (4) holds,

$$\delta^2 \quad > \quad (1 - \delta(1 - \delta))z_j + \delta z_i$$

and, if the optimal offer is rejected, then the aspirations satisfy (4). That is

$$\left(\delta + (\delta(\delta - 1) - 1)\left(\frac{\delta^2}{\delta^2 + 1} - 1 \right) \right) z_i + \left(-(\delta(\delta - 1) - 1)\left(\frac{\delta^2}{\delta^2 + 1} - 1 \right) \right) z_j$$
$$\geq \left(\delta^2 - \left(\frac{\delta}{\delta^2 + 1} - 1 \right)(\delta(\delta - 1) - 1) \right)$$

where the expression follows from plugging in (4) the transition rules z'_r and z'_p into z_p and z_r, respectively, and the optimal proposal into the responder transition rule and simplifying.

Suppose now that when that for $k > 2$, the set I^k is constrained by

(5) $C_{k-1} > B_{k-1}z_r + A_{k-1}z_{p'}$,

and by

(6) $C_k \le A_k z_r + B_k z_p$

for some constants $A_{k-1}, A_k, B_{k-1}, B_k, C_{k-1}$ and C_k. Here z_p and z_r are the aspirations of the proposer and the responder when $z \in I^k$, respectively, and suppose that the optimal proposal is

(7) $\widehat{c}^k = \dfrac{O_k + \delta^k(a_{k-1}z_r - a_k z_p)}{N_k}$

where $O_2 = \delta$, $N_2 = 1+\delta^2$, $N_k = 1+\sum_{l=1}^{k-1} a_l\delta^{l+1}$, $O_k = \delta N_{k-1}-\delta O_{k-1}+a_{k-1}\delta^k$ and $\{a_l\}_{l=1}^{\infty}$ is the Fibonacci sequence.

Let us show that then

(8) $\widehat{c}^{k+1} = \dfrac{O_{k+1} + \delta^{k+1}(a_k z_r - a_{k+1}z_p)}{N_{k+1}}$

and that I^{k+1} is characterized by

(9) $C_k > A_k z_p + B_k z_r$

and

$$C_k + B_k(\frac{1}{N_k}O_k - 1)$$
$$\le \quad (A_k + B_k(\frac{\delta^k}{N_k}a_k - 1))z_p$$
$$+ B_k(1 - \frac{\delta^k}{N_k}a_{k-1})z_r$$

But (9) follows from the fact that, if $C_k \le A_k z_p + B_k z_r$ held, then $z \in I^k$ and $z \notin I^{k+1}$. Therefore, when $z \in I^{k+1}$, $C_k > A_k z_p + B_k z_r$. On the other hand, given the transition rules, if the optimal proposal when $z \in I^{k+1}$ is rejected, then it must be that t $z \in I^k$. Thus,

(10) $C_k \le A_k z_p + B_k(z_r + 1 - z_p - \widehat{c}^{k+1})$.

The optimal proposal makes r indifferent between accepting and rejecting. That is, $\widehat{c}^{k+1} = \delta(1 - \widehat{c}^k)$. The responder at the round before strictly prefers accepting any higher offer and rejecting any smaller offer since, due to reciprocal aspirations, the optimal proposal at the latter round is increasing in the proposal at the former. On the other hand the proposer at the former strictly prefers $1-\widehat{c}^{k+1}$ to $\delta\widehat{c}^k$. Thus, she strictly prefers proposing $\widehat{c}^{k+1}$ to proposing any other offer. Thus, $\widehat{c}^{k+1} = \delta(1 - \widehat{c}^k)$ gives the optimal offer at the former round.

$\widehat{c}^{k+1}$

$$= \delta(1 - \widehat{c}^k)$$

$$= \delta(1 - \frac{O_k + \delta^k(a_{k-1}z'_r - a_k z'_p)}{N_k})$$

$$= \delta(1 - \frac{O_k + \delta^k(a_{k-1}z_p - a_k(z_r + 1 - z_p - \widehat{c}^{k+1}))}{N_k})$$

where z' describe the aspirations at the latter round and z at the former. Simplifying gives the desired expression. Moreover, plugging in an analogous manner the expression of $\widehat{c}^{k+1}$ into (10) and simplifying gives the remaining inequality constraint in I^{k+1}. Thus,

$$A_{k+1} = B_k(1 - \frac{\delta^k}{N_k}a_{k-1})$$

$$B_{k+1} = A_k + B_k(\frac{\delta^k}{N_k}a_k - 1)$$

$$C_{k+1} = C_k + B_k(\frac{O_k}{N_k} - 1)$$

Let us next show that as players become infinitely patient, the first round equilibrium proposal tends to the golden sharing.

Notice first that $lim_{\delta \longrightarrow 1} N_k(\delta) = a_{k+1}$. Second, $lim_{\delta \longrightarrow 1} O_3 = N_2 - O_2 + a_2 = a_3$, and recursively, $lim_{\delta \longrightarrow 1} O_k = N_{k-1} - a_{k-1} + a_{k-1} = a_k$.

Thus,

(11) $lim_{\delta \longrightarrow 1} \dfrac{O_{k+1}(\delta)}{N_{k+1}(\delta)} = \dfrac{a_k}{a_{k+1}}$

and thus $lim_{k \longrightarrow \infty} lim_{\delta \longrightarrow 1} \widehat{c}^k = 1/\varphi$.

To see that the scope for negotiations at the first round approaches infinity as $\delta \longrightarrow 1$, notice that

$$lim_{\delta \longrightarrow 1} A_{k+1} = B_k(1 - \frac{a_{k-1}}{a_{k+1}})$$

$$lim_{\delta \longrightarrow 1} B_{k+1} = A_k + B_k(\frac{a_k}{a_{k+1}} - 1)$$

$$lim_{\delta \longrightarrow 1} C_{k+1} = C_k + B_k(\frac{a_k}{a_{k+1}} - 1)$$

since given that $lim_{\delta \longrightarrow 1} B_2 = lim_{\delta \longrightarrow 1} A_2 = lim_{\delta \longrightarrow 1} C_2 = 1/a_3$ and that $\frac{a_k}{a_{k+1}} -$ and that $1 = -\frac{a_{k-1}}{a_{k+1}}$ by the definition of the Fibonacci sequence,

$$lim_{\delta \longrightarrow 1} A_k = B_k = C_k = \frac{1}{a_3 a_k}$$

Thus, for a given k the inequality constraint is of the form $v_k \le v_k z_p + v_k z_r$, where $v_k > 0$ and $z_p, z_p \in [0, 1]$. Therefore the constraint is satisfied for no k implying that, as δ tends to 1, the scope for negotiations tends to infinity.

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