1 IAM set up

1.1 Climate change

For our climate change dynamics, we use a multi-box model for atmospheric CO2 stocks, as a convenient approximation of various processes including CO2 storage in oceans and the biosphere. We model temperature adjustment through a one-box system. Damages are typically assumed proportional to the quadratic of the temperature change, while the temperature depends on the logarithm of atmospheric concentrations. It has been noted that this combination of a convex damage function and concave response function returns an almost linear relation between atmospheric CO2 and damages (see paper for analysis). Let there be multiple atmospheric CO2 boxes, labeled i, such that a share a_i of antropogenic emissions enter box i, $\sum_i a_i = 1$, while the depreciation rate of box i is β_i. The stock S_i in box i develops according to

$$\begin{aligned} S_{i,t} &= (1-\eta_i)S_{i,t-1} + a_i z_{t-1} \\ S_t &= \sum\nolimits_i S_{i,t} \end{aligned}$$

where z_t are emissions, for which we use fossil fuel energy use as a proxy. Given the atmospheric CO2 stock, the temperature slowly adjustes, also according to a multi-box system. We use the variable $D_{j,t}$ as a proxy for squared temperature. Let π be the sensitivity of (squared) temperature for atmospheric CO2, so that if S_t is constant then $D_t = \pi S_t$ follows, and γ_j is the adjustment speed of the $j-th$ box, $\sum_j b_j = 1$:

$$\begin{aligned} D_{j,t} &= D_{j,t-1} + \varepsilon_j (b_j \pi S_t - D_{j,t-1}) \\ D_t &= \sum\nolimits_j D_{j,t} \end{aligned}$$

Notice that in this formulation, part ε_j of the temperature change is immediate. Given these two layers of climate variables, it is a straightforward matter of verification that future damages depend on past emissions linearly as:

$$\begin{aligned} S_{i,t} &= \sum\nolimits_{\tau=0}^{\infty} a_i (1-\eta_i)^{\tau} E_{t-\tau-1} \\ D_{j,t} &= \sum\nolimits_i \sum\nolimits_{\tau=1}^{\infty} a_i b_j \pi \varepsilon_j \frac{(1-\delta_i)^{\tau+1} - (1-\varepsilon_j)^{\tau+1}}{\varepsilon_j - \delta_i} E_{t-\tau-1} \end{aligned}$$

1.2 Utility and Technology

The basic model only considers utility from the consumption of goods. For convenience we consider a logarithmic utility function

$$u_t = \ln(c_t / l_t) - \Delta_u D_t$$

where population l_t follows an exogenous path. Welfare is given by discounted utility, formulated recursively as

$$\begin{aligned} w_t &= u_t + \beta \delta W_{t+1} \\ W_t &= u_t + \delta W_{t+1} \end{aligned}$$

In the final period t_L, next-period welfare is defined through

$$W_{t_L+1} = \xi_k \ln(k_{t_L+1}) - \sum \xi_{S_i} S_{i,t+1} - \sum \xi_{D_j} D_{j,t+1}$$

where $\xi_k = \frac{\alpha}{1-\alpha\theta}$, and the ξ_{S_i} and ξ_{D_j} are defined by

$$\begin{aligned}
\xi_{D_j} &= \Delta \textstyle\sum_{\tau=0}^{\infty} \delta^\tau (1-\varepsilon_j)^\tau = \dfrac{\Delta_u + \Delta_y/(1-\alpha\delta)}{1-\delta(1-\varepsilon_j)} \\
\xi_{S_i} &= \textstyle\sum_j \sum_{\tau=0}^{\infty} \delta^{\tau+1} (1-\delta_i)^{\tau+1} b_j \pi \varepsilon_j \xi_{D_j} = \sum_j \dfrac{b_j \pi \varepsilon_j \xi_{D_j} \delta(1-\eta_i)}{1-\delta(1-\eta_i)} \\
\zeta_1 &= \textstyle\sum_i \left(a_i \xi_{S_i} + \sum_j b_j \pi \varepsilon_j \xi_{D_j} \right) = \sum_j \dfrac{b_j \pi \varepsilon_j \xi_{D_j}}{1-\delta(1-\eta_i)}
\end{aligned}$$

Output y_t is produced using a Cobb-Douglas technology, overall productivity $A_{y,t}$, capital k_t, labour inputs $l_{y,t}$, energy inputs e_t, and an adjustment factor for climate change damages $\omega(s_t)$.

$$y_t = \omega(s_t) k_t^\alpha [A_t(l_{y,t}, e_t)]^{1-\alpha} \tag{1}$$

$$A_t(l_{y,t}, e_t) = \min\{A_{y,t} l_{y,t}, A_{e,t} e_t\} \tag{2}$$

where we notice that, in equilibrium, for strictly positive labour and energy prices the minimum condition is equivalent to two equations $A_t(l_{y,t}, e_t) = A_{y,t} l_{y,t} = A_{e,t} e_t$.

Energy can be produced using fossil fuels, $e_{f,t}$, or using carbon-free (emission-neutral) technologies, $e_{n,t}$.

$$e_t = e_{f,t} + e_{n,t}$$

Production of fossil fuel-based energy requires labour in proportional amounts $l_{f,t}$, and fossil fuels z_t, where coefficients $A_{f,t}$ and B_t describe productivity of labour and fossil fuels, respectively. Fossil fuel-based energy, as an industrial process, has constant returns to scale.

$$e_{f,t} = \min\{A_{f,t} l_{f,t}, B_t z_t\} \tag{3}$$

Carbon free energy has decreasing returns to scale, as it is land-intensive, with for given wages, an elasticity of supply $\varphi > 0$.

$$e_{n,t} = \frac{\varphi+1}{\varphi} (A_{n,t} l_{n,t})^{\frac{\varphi}{\varphi+1}} \tag{4}$$

The labour market clears

$$l_{y,t} + l_{f,t} + l_{n,t} = l_t$$

while climate change damages satisfy

$$\begin{aligned}
\omega(s_t) &= \exp(-\Delta_y D_t) && (5) \\
&= \exp(-\Delta_y \sum_i \sum_j \sum_{\tau=1..} a_i b_j \pi \frac{(1-\eta_i)^{\tau+1} - (1-\varepsilon_j)^{\tau+1}}{\varepsilon_j - \eta_i} z_{t-\tau}), && (6)
\end{aligned}$$

We consider the term between brackets as a proxy for the global mean (surface) temperature (GMT_t) squared, so that we can ex-post calculate the temperature path as

$$GMT_t = \sqrt{D_t}$$

1.3 Equilibrium

From the analysis, we have the investment share as

$$g = \frac{k_{t+1}}{y_t} = \frac{\alpha\rho}{1 + \alpha(\rho - \theta)} \tag{7}$$

and the first order conditions for labour. Let us use p_t as the shadow price for the consumer good, w_t as the shadow price for labour, and q_t as the shadow price for energy. The shadow price for the temperature proxy D_t is ξ_D, but this plays no role in the FOCs. The emissions shadow price is $\beta\delta\zeta_1$ where

$$\begin{aligned}\zeta_1 &= \textstyle\sum_i \sum_j \frac{a_i b_j \pi \varepsilon_j [\Delta_u + \Delta_y/(1-\alpha\theta)]}{[1-\theta(1-\delta_i)][1-\theta(1-\varepsilon_j)]} \\ &= \textstyle\sum_i \frac{a_i \xi_{S_i}}{\delta(1-\eta_i)}\end{aligned}$$

. The first-order conditions for consumption and the final goods sector give

$$p_t = \frac{1}{(1-g)y_t} \tag{8}$$

$$w_t l_{y,t} + q_t e_t = (1-\alpha) p_t y_t \tag{9}$$

Fossil fuel use is positive if

$$q_t = \frac{w_t}{A_{f,t}} + \frac{\beta\delta\zeta_1}{B_t} \tag{10}$$

and zero if the left-hand-side is stricly smaller.

The carbon-free energy source supply is given by the first order condition

$$q_t = w_t \frac{\partial l_{n,t}}{\partial e_{n,t}} = \frac{w_t}{(A_{n,t})^{\frac{\varphi}{\varphi+1}}} (l_{n,t})^{\frac{1}{\varphi+1}} \tag{11}$$

Through substitution, we can determine the allocation of labour for final goods and energy production within each period, seperately, as four equations in four unknowns $l_{y,t}, l_{f,t}, l_{n,t}, w_t$:

$$A_{y,t} l_{y,t} = A_{e,t}\left(A_{f,t} l_{f,t} + \frac{\varphi+1}{\varphi}(A_{n,t} l_{n,t})^{\frac{\varphi}{\varphi+1}}\right) \tag{12}$$

$$w_t l_t + \frac{\rho\zeta_1 A_{f,t}}{B_t} l_{f,t} + \frac{1}{\varphi} w_t l_{n,t} = \frac{1-\alpha}{1-g} \tag{13}$$

$$\frac{w_t}{A_{f,t}} + \frac{\rho\zeta_1}{B_t} \geq \frac{w_t}{(A_{n,t})^{\frac{\varphi}{\varphi+1}}} (l_{n,t})^{\frac{1}{\varphi+1}} \perp l_{f,t} \geq 0 \tag{14}$$

$$l_{y,t} + l_{f,t} + l_{n,t} = l_t \tag{15}$$

The first equation represents the production function (1) and (2). The second equation allocates the value of output that is not attribted to capital (the right-hand side) to the factors labour, carbon emissions, and space rent for non-carbon energy. The third equation compares the production costs for fossil fuel energy with non-carbon energy, and the last equation adds all labour use.

We note that for the BAU scenario, where we abstract from climate externalities, we set $\zeta_1 = 0$, and we can immediately derive $l_{n,t}$ from the third equation, then we get w_t from the second equation, and we can easily derive $l_{y,t}$ and $l_{f,t}$ from the first and last equation.

$$
\begin{aligned}
l_{n,t} &= \frac{A_{n,t}^{\varphi}}{A_{f,t}^{\varphi+1}} \\
w_t &= \frac{1-\alpha}{1-g}\frac{\varphi}{\varphi l_t + l_{n,t}} \\
l_{y,t} &= \frac{A_{e,t}}{A_{y,t} + A_{e,t}A_{f,t}}[A_{f,t}(l_t - l_{n,t}) + \frac{\varphi+1}{\varphi}(A_{n,t}l_{n,t})^{\frac{\varphi}{\varphi+1}}] \\
l_{f,t} &= l_t - l_{y,t} - l_{n,t}
\end{aligned}
$$

For $\zeta_1 = A_{n,t} = 0$, there are no climate externalities and no carbon-free energy, and labour is divided between the final goods sector and the energy sector in shares proportional to productivity:

$$
\begin{aligned}
l_{n,t} &= 0 \\
l_{y,t} &= \frac{A_{f,t}A_{e,t}}{A_{f,t}A_{e,t} + A_{y,t}} l_t \\
l_{f,t} &= \frac{A_{y,t}}{A_{f,t}A_{e,t} + A_{y,t}} l_t \\
w_t &= \frac{1-\alpha}{(1-g)l_t}
\end{aligned}
$$

2 Calibration

2.1 Climate parameters

The file 'Calibration vx.xls' uses as inputs parameters from the literature, and calculates from these the parameters as used by the simulation model. It has two tab sheets. The first tab sheet calibrates the carbon cycle model. It's inputs are data on emissions and estimates from the literature regarding the stocks of CO2 in various layers. Its output is an estimate of the CO2 stocks in the various boxes, and the shares and depreciation within the boxes:

$$
\begin{aligned}
S_{t=2005} &= (.309, .265, .236) \\
a &= (.166, .205, .429) \\
\eta &= (0, .088, .493).
\end{aligned}
$$

Temperature dynamics are taken straight from the literature. We assume a 1-box damage model and choose the parameters as follows:

$$b = 1, \varepsilon = .183, \pi = 4.23$$

We thus have one box ($b = 1$). We inteprete D_t in (**??**) as the Global Mean Temperature (GMT) squared ($GMT_t = \sqrt{D_t}$); $\varepsilon = .183$ in the decadal model implies a temperature adjustment speed of 2 per cent per year. Choice $\pi = 4.23$ $\left[K^2/GtCO_2\right]$ implies a climate sensitivity of 3 $Kelvin$ per $2.129TtCO_2$. These choices are within the ranges of scientific evidence (Solomon et al. 2007).

We seek to calibrate the damage parameters to match the case presented in Nordhaus (2007) as a benchmark. Assuming damages equivalent to 2.7 per cent of output at a temperature rise of 3 Kelvin, as in Nordhaus (2001), we obtain $\Delta_y = 0.003$; we set $\Delta_u = 0$, unless otherwise stated.

2.2 Population

Population is assumed to follow a logistic growth curve:

$$L_{t+1} = [1 + \gamma_L(1 - \frac{L_t}{L_{\max}})]L_t$$

with parameters given by World Bank forecasts. Population in 2010 (L) is set at 6.9 [billion], while the maximum population growth rate γ_L is chosen such that in 2010 the effective population growth rate per decade equals 0.12 [/decade]. The maximum expected population (reached at about 2200) is set at 11 [billion].

2.3 Economic parameters

The second tab sheet of the file 'Calibration vx.xls' calibrates the economic model. Capital elasticity α follows from the assumed time-preference structure β and δ, and observed historic gross savings g, through (7). As a base-case, we consider net savings of 25% ($g = 0.25$), and a 2 per cent annual pure rate of time preference ($\beta = 1, \delta = 0.817$), resulting in $\alpha = g/\delta = 0.306$.

The economic parameters are based on a benchmark scenario path (a 'business as usual') where for the calibration part, climate damages are set to zero, $\omega(.) = 1$, $\zeta_1 = 0$. For this counterfactual zero-damages benchmark path, we extrapolate past historic trends to construct a path with values for population l_t, output y_t, fossil fuel and carbon-free energy supply $e_{f,t}$ and $e_{n,t}$, associated emissions z_t, and energy prices relative to the final good q_t/p_t. Given the path $(l_t, y_t, e_{f,t}, e_{n,t}, q_t/p_t)$, we derive the path for parameters $(A_{y,t}, A_{e,t}, A_{f,t}, A_{n,t})$ using the following constructive procedure.

First, through the savings rule (7), we determine all k_t.[1] From the FOC (8) we derive p_t. This also provides the value for q_t. we can rewrite the FOC (11)

[1] We need also the output of the period before the calibration horizon, y_0, to determine k_1.

as

$$w_t l_{n,t} = \frac{\varphi}{\varphi + 1} q_t e_{n,t}$$

to derive the value for $w_t l_{n,t}$, which, when plugged in (13) gives us w_t. Together with the previously calculated value for $w_t l_{n,t}$, we now have $l_{n,t}$, and through (4) or (11), we derive $A_{n,t}$. Through the FOC (10), where $\zeta_1 = 0$, we derive $A_{f,t}$, and through (3) we get $l_{f,t}$. The labour balance (15) gives now $l_{y,t}$. The production function (1) and (2) now provide us with $A_{y,t}$ and $A_{e,t}$. This completes the calculation of the productivity parameters $(A_{y,t}, A_{e,t}, A_{f,t}, A_{n,t})$. Energy production is measured in CO2-emission equivalents, so that by construction $B_t = 1$. The resulting paths for technology are put into a text-file (productivity.inc) that is read into the GAMS code.

The basic assumption for the benchmark path is that per capita output grows at a rate of 1.5% per year, starting at y_t=600 [Teuro/decade] in 2010, initially, converging to a long-term per-capita output level of 10 times the 2010 values. Emissions are calibrated at z_t=0.306 [TCO_2/decade].

Data used for calibration:

Emissions 2010 (E): 0.306 [TtCO2/decade]

price of energy 2010 (average for coal and oil) (q) : 50 [Teuro/TtCO2]

Gross VA 2010 (Y): 77 trillion USD/yr in 2010 (WB, using PPP) = 60 Teuro/yr = 600 [Teuro/decade] for the first decade 2006-2015. Inflation 1990-2010 was about 1.71, so that it equals 45 trillion 1990USD/yr.

Fossil fuel energy ($e_{f,t}$): 0.306 [TtCO2/decade], with $e_{f,t} + e_{n,t}$ increasing at the population growth rate plus half of the per-capita output growth rate.

Commercial carbon-free energy: ($e_{n,t}$): set at 10% of $e_{f,t}$, in 2010 (SRES), and its share increasing by 1% per year. That is, the observed rapid growth of the last decade in carbon-free energy is considered a consequence of implicit climate policy, not driven by fundemental changes in the costs of renewables versus fossil fuels.

Net savings (s): 0.30

Time preference when consistent: $\beta = 1, \delta = 0.817$ [/decade] (2% discount rate per year)

Economic growth (g_Y): 1% per capita per year.

3 Results

The simulation model is run in GAMS-IDE, and writes directly output to an Excel file. This file is then copied into an excel file that converts the numbers into the figures used for the paper.