Strategic Resource Dependence

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Abstract

We consider a situation where an exhaustible-resource seller faces demand from a buyer who has a substitute but there is a time-to-build delay for the substitute. We find that in this simple framework the basic implications of the Hotelling model (1931) are reversed: over time the stock declines but supplies increase up to the point where the buyer decides to switch. Under such a threat of demand change, the supply does not reflect the current resource scarcity but it compensates the buyer for delaying the transition to the substitute. The analysis suggests a perspective on costs of oil dependence.

JEL Classification: D4; D9; O33; Q40.

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1 Introduction

Policies such as fuel taxes, technology programs, or even international agreements on pollution emissions reductions are likely to entail a demand change in some important

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exhaustible-resource markets. When resource sellers are strategic, they have an incentive to distort these policies to their own advantage, potentially leading to an increased dependence on the resource. To understand the seller's effort to distort the adoption of demand-changing policies, we consider a simple framework where a monopolistic seller (or a group of sellers coordinating actions) of an exhaustible resource faces demand from a buyer (or a group of buyers coordinating actions) who has a substitute but there is a time-to-build delay for the substitute. We find that in this framework the basic implications of the Hotelling model (1931) are reversed: over time the resource stock declines but supplies increase, rather than decrease, up to the point where the buyer decides to initiate the transition to the substitute. Under such a threat of change in the demand infrastructure, the supply today does not reflect the true resource scarcity, but it seeks to postpone the buyer's decision by compensating for the future scarcity felt during the transition time to the substitute when the buyer is still dependent on the resource.

Our research builds on Hotelling's theory of exhaustible-resource consumption (1931), Nordhaus' (1973) concept of a backstop technology,[1] and the extensive literature on strategic equilibria in resource economics. Our main addition to the standard framework for analysis is the inclusion of a time-to-build delay for the backstop. Previous literature closest to our approach can be divided on the assumptions made for the strategic variable on the buyer side.[2] First, there is a large literature on optimal tariffs in depletable-resource markets showing how coordinated action on the buyer side can be used to decrease the seller's resource rent (e.g., Newbery, 1983, Maskin and Newbery, 1990; see Karp and Newbery 1993 for a review). Hörner and Kamien (2004) provide a general view on these models by showing that the problem faced by a monopsonistic exhaustible-resource buyer is formally equivalent to that faced by a Coasian durable-

[1]Nordhaus (1973) was the first to define and analyze the concept of backstop technology in exhaustible-resource markets. He defined it as follows: "The concept that is relevant to this problem is the *backstop technology*, a set of processes that (1) is capable of meeting the demand requirements and (2) has a virtually infinite resource base" (Nordhaus, 1973, pp. 547-548).

[2]There is a large but less closely related literature focusing purely on seller power in the exhaustible-resource framework. Hotelling himself (1931) already analyzed the monopoly case. Salant (1976) considered an oligopolistic market structure with one dominant firm, and Lewis and Schmalensee (1980) analyzed an oligopoly where all firms have some market power. This literature has developed on two frontiers. First, it has focused on developing less restrictive production strategies: from path strategies as in Lewis and Schmalensee, Loury (1986) and Polansky (1992), to decision rule strategies as, for example, in Salo and Tahvonen (2001). Second, the literature has developed more natural cost concepts for extraction under which the resource is economically rather than physically depleted. See Salo and Tahvonen (2001) for a discussion and contribution on this.

good monopoly. We depart from the Coasian framework because the buyer is not a pure monopsony and has a different strategic variable (the substitute). While import tariffs and fuel taxes are important, they are more flexible instruments as compared to the development or adoption of substitute technologies that have a permanent effect on the resource dependence. To be effective, optimal tariffs have to be successful in changing the dynamic demand perceived by the seller. The degree of success obviously depends on the precise formulation of the game, but generally the seller's sales path still follows a Hotelling rule modified to take into account the buyers' tariff policy. This leads to supplies declining over time. We believe that the technology threat potentially is a more important determinant of how sellers perceive their future demand. It thus creates potentially greater or at least very different strategic threats to the seller. [3]

Second, there is a large but somewhat dated literature on the same bilateral monopoly situation where the buyers' strategic variable is to develop or adopt a substitute technology. Early papers such as Dasgupta *et al.* (1983), Gallini *et al.* (1983), and Hoel (1983) assume the buyer exploits a Stackelberg leadership and can commit to a deterministic R&D program for the development of the substitute. The results provide interesting insights into how the buyer side can extract the seller's rent by altering the timing of sales. Later developments analyzed the role of leadership and commitment (Lewis *et al.*, 1986) and, finally, probabilistic success in R&D and Markov-perfect strategies (Harris and Vickers, 1995). None of the above papers predict that the basic Hotelling implications are reversed, although Harris and Vickers (1995) obtain a result that sales path may be non-monotonic (but not generically increasing).[4]

The market structure we describe is such that not only sellers have market power but also buyers enjoy some power so that no party is in complete leadership. The nature of the strategic interaction between buyers and sellers is preserved in the limiting case without discounting, which allows an essentially static analysis and it shows the way

[3]The idea that resource supply strongly reacts to perceived future technological options is central in the green paradox literature. This literature uses the standard Hotelling model and concludes that more optimistic expectations on the arrival of an oil substitute will increase current oil supply, and thereby worsen climate change (Hoel 2008, Sinn 2008). See Gerlagh (2010) for a discussion of some assumptions in this literature.

[4]It should be clear that we are focusing on how strategic relationships in the resource market shape the supplies. There are also other ways to explain the failure of the standard Hotelling model (see Dasgupta and Heal (1974) for the standard model), or its extensions, to match reality (see Krautkramer (1999) for a review of the literature). And there are other ways to extend the traditional economic growth-resource depletion model such that supplies increase over time (cf. Tahvonen and Salo 2001).

to analyze the discounted case. Moreover, in addition to the different market structure assumptions, we depart from previous literature in that we abstract from the precise instrument implementing the structural change in demand: when action is taken, it changes the demand irreversibly after a time lag. This abstraction simplifies the strategic variable on the buyer side while keeping what seems essential in the relationship.

The structure of the paper is the following. In Section 2, we discuss some developments in the oil market that motivate our study. In Section 3, we introduce the basic resource allocation problem by considering the social optimum, consumers' optimum, and also by having a first look at the equilibrium. In Section 4, we introduce and analyze the equilibrium without discounting. In Section 5, we investigate the changes to equilibrium and robustness of overall findings under discounting. In Section 6, we conclude by discussing alternative approaches to the problem and potential implications for the oil market.

2 Motivating example: the market for cheap oil

Our contribution is to the basic exhaustible-resource theory but we are motivated by some recent developments affecting the oil market. First, while there is no single buyer in the oil market, policies aiming to reduce dependence on imported oil imply a collective action on the consumer side. Whatever the reason for policies – need to safeguard the economy against macroeconomic risks or perhaps global warming – they are likely to affect how oil producers perceive their future demand, influencing supplies today.[5] The results suggest that, under such a threat of structural change in oil demand, the true resource scarcity cannot be read from the current supply.

Second, while it is clear that the world will never run out of all fossil fuel sources, it is equally clear that we may run out of conventional, cheap oil. The ownership of the cheapest oil reserve is extremely concentrated by any measure and concentration is expected to increase in the near future.[6] The concentration of ownership implies that strategic management of the cheap oil stocks is likely even without a formal cartel among producers. Cheap oil producers understand their influence on market development

[5] The Stern Review on the Economics of Climate Change (2006), while being a very comprehensive cost-benefit analysis, is also a political document illustrating the willingness to take actions changing the demand for fossil-fuels.

[6] See the "2007 Medium-Term Oil Market Report" published by the International Energy Agency for estimates of the Core OPEC reserves. The Saudi share of the Core OPEC stocks is expected to increase over time.

and take an active role in "demand management"; they often communicate like central bankers with the market, emphasizing credibility and security of supply.[7] The resource that, for example, Saudi Arabia is controlling is unique in that it allows extraction of high quality output with relatively little capital investment. It also allows for rapid and large production rate changes. Reserves with such properties are at the heart of the economics of the oil dependence because, roughly put, the remainder of the fossil fuel supply is capital intensive and costly when used for the production of liquid fuels. In fact, what is essential for the strategic interaction that we consider is the existence of a low-cost but finite reserve with concentrated ownership and inelastic short-run demand; the rest of 'oil' production can be seen as part of substitute fuel production, including costly conventional oil sources, nonconventional oils, biofuels, and alternative energy sources.[8]

While the relationship between major oil importers and exporters is clearly not an open bargaining situation, as explicit contracts are not conceivable in the context, it has a flavor of bargaining taking place through markets where offers and responses are implicit. Sellers' focus on secure supply suggests a compensation to the importing party for continuing potentially costly dependence. On the buyer side, trust in the relationship is expressed by voluntary inaction, that is, postponement of actions changing the demand structure. Our timing assumptions for strategies are perhaps better suited for capturing what is material in this kind of relationship than those used in earlier literature.

3 The resource allocation problem

There are two agents, the buyer and the seller of an exhaustible resource. The buyer's flow payoff from consuming the resource is given by the function

$$u : \mathbb{R}_+ \mapsto \mathbb{R}_+$$

[7]The following citation describes this: "We've got almost 30 percent of the world's oil. For us, the objective is to assure that oil remains an economically competitive source of energy. Oil prices that are too high reduce demand growth for oil and encourage the development of alternative energy sources" (Adel al-Jubeir, foreign policy adviser of crown prince Abdullah of Saudi Arabia, Herald Tribune, Jan 24, 2007).

[8]There are different definitions of conventional and nonconventional oils, and these also change over time; see the Hirsch Report (prepared for the U.S. Department of Energy, 2005). The report emphasizes that the important scarcity is in the reserves of high-quality conventional oil.

where $u(q)$ is assumed to be increasing, continuously differentiable, and nonlinear everywhere. The seller's flow payoff from selling the resource is given by the function

$$\pi : \mathbb{R}_+ \mapsto \mathbb{R}_+$$

where $\pi(q)$ assumed to be strictly concave and continuously differentiable in q. Payoffs u and π are connected through a strictly concave consumer's utility function

$$\tilde{u} : \mathbb{R}_+ \mapsto \mathbb{R}$$

such that $u(q) = \tilde{u}(q) - \tilde{u}'(q)q$ and $\pi(q) = \tilde{u}'(q)q$. The consumer price is thus $p_t = \psi(q_t) = \widetilde{u}'(q_t)$, and demand is defined by $q_t = D(p_t) = \psi^{-1}(p_t)$.

The buyer is thus an agent (e.g., government) whose flow payoff is the consumer surplus $u(q)$. The seller is a resource monopoly. The initial resource endowment s_0 is finite. Time is continuous and consumption depletes the stock at rate q_t. We assume no extraction costs. There is a substitute for the resource that ends the need to use the resource. The buyer can choose to adopt the substitute at any t, and then wait for interval of time k, so that the alternative supply infrastructure arrives at time $t + k$ and provides a surplus flow $\bar{u}$ to consumers thereafter. Thus, after the time-to-build delay, the substitute fully replaces the resource: by assumption, the resource is not needed after the change.[9]

The economy can thus be in one of the three main regimes, denoted by $\mathcal{S} \in \{C, I, L\}$. Regime C is the continuation regime that prevails if and only if the buyer has not chosen to end the relationship in the past. Regime I is the interim regime where the buyer has already made the stopping decision but the substitute for the seller's supply has not yet arrived. We assume that state I lasts k units of continuous time. Regime L is the long-run regime where the substitute is in place. Thus, the adoption of (or investment in) the substitute causes the transition from C to I and, after k units of time, from I to L.[10]

For interpretation, we can assume cost flow c for maintaining the alternative supply infrastructure and define the long-run surplus flow as

$$\bar{u} = \widetilde{u}(D(0)) - c,$$

[9]We can relax this assumption, without changing the main result, by letting the resource compete with the substitute, or by making the change gradual and uncertain. We discuss these extensions after the main model in Section 4.4 and in Appendix 9

[10]We will refer to the adoption of and investment in the substitute interchangeably.

but this interpretation is not necessary for the model, i.e., $\bar{u}$ need not be linked to the original utility formulation and then we can abstract from cost flow c.

We assume no discounting.[11] We denote the seller's stock-dependent payoff by $V(s_t)$ and consumers' payoff by $W(s_t)$ if there has been no investment before t. Expression $V(s_t)$ measures cumulative (undiscounted) future profits while $W(s_t)$ measures cumulative surplus from the excursion above the long-run surplus from time t onwards:

$$V(s_t) = \int_t^{T+k} \pi(q_\tau)d\tau, \tag{1}$$

$$W(s_t) = \int_t^{T+k} [u(q_\tau) - \overline{u}]d\tau, \tag{2}$$

where T is the adoption time for the substitute, and $T+k$ is the arrival time for the substitute. The social optimum determines the time interval of resource use, $T+k$, and the supply path q_t, that maximizes total resource surplus

$$\mathcal{W}(s_t) = V(s_t) + W(s_t) = \int_t^{T+k} [\widetilde{u}(q_\tau) - \overline{u}]d\tau. \tag{3}$$

3.1 Socially optimal resource dependence

Consider now the following simple question: how much of the resource should be used before actions are taken, and how much should be left for the transition time interval towards the substitute?

The socially optimal supply solves a simple problem. Let

$$\mathcal{W}^I(s_T) = \int_T^{T+k} [\widetilde{u}(q_\tau) - \overline{u}]d\tau$$

be the optimal social surplus at the time of investment T subject to $ds_\tau/d\tau = -q_\tau$ and s_T given. Since $\widetilde{u}(q_\tau)$ is strictly concave and there is no discounting, the optimal path is constant at level $q_\tau = s_T/k$ for $\tau \in [T, T+k]$. This defines the marginal value of the resource stock as

$$\mathcal{W}^{I\prime}(s_T) = \widetilde{u}'(s_T/k). \tag{4}$$

The optimal supply over $[t, T]$ solves

$$\max_{\{q_\tau, T\}} \int_t^T [\widetilde{u}(q_\tau) - \overline{u}]d\tau + \mathcal{W}^I(s_T).$$

[11] In Section 5, we extend the model to positive discounting. It is not obvious that the undiscounted case is the true discounted equilibrium limit (see Dutta 1991), but in our case it is, as we will verify.

The current-value Hamiltonian is $\mathcal{H} = \widetilde{u}(q_\tau) - \overline{u} - \lambda_\tau q_\tau$, and the interior first-order conditions are

$$\begin{aligned} \mathcal{H}_q &= \widetilde{u}'(q_\tau) - \lambda_\tau = 0 & (5) \\ d\lambda_\tau/d\tau &= 0, \lambda_T = \mathcal{W}^{I\prime}(s_T) & (6) \\ \mathcal{H}(T) &= \widetilde{u}(q_T) - \overline{u} - \lambda_T q_T = 0. & (7) \end{aligned}$$

The last condition follows since $\mathcal{W}^I(s_T)$ does not depend on T directly. Let q^{**} denote the socially optimal consumption path.[12] Combining (5)-(7) implies that q^{**} is constant throughout, and given by $q^{**} = s_0/k$ if $T = 0$, and by

$$\widetilde{u}(q^{**}) = \overline{u} + q^{**}\widetilde{u}'(q^{**}) \tag{8}$$

otherwise. Since $\widetilde{u}(q)$ is strictly concave, s_0 large enough implies that $T > 0$, which is assumed throughout. Since the consumer surplus is $u(q) = \widetilde{u}(q) - q\widetilde{u}'(q)$, we must have

$$u(q^{**}) = \overline{u}. \tag{9}$$

From another angle, we can see that the optimal policy maximizes λ, or equivalently, the average excursion of utility above the long-run utility $\overline{u}$:

$$\lambda = \max_q [\widetilde{u}(q) - \overline{u}]/q. \tag{10}$$

It is instructive to see Figure 1, where we can find the social optimal supply level $q = q^{**}$ on the curve of utility $\widetilde{u}(q)$ such that the line through $(0, \overline{u})$ and $(q, \widetilde{u}(q))$ has the steepest slope.

Proposition 1 *In the social optimum, consumers receive reservation utility level $\overline{u}$ in all regimes, while producers receive all the resource surplus. Consumers do not benefit from an increase in the resource stock, $W'(s_0) = 0$.*

Proof. The first part of the proposition states that along the social optimal path, the buyer side is indifferent between resource dependence and the substitute technology. This part follows immediately from (9). The last part of the proposition then follows from the definition of the buyer's payoff (2). ■

Note that the optimal consumption rate determines the overall time span of resource dependence by $T + k = s_0/q^{**}$. The investment date T in turn is determined by the requirement that $q_t = q^{**}$ before and after the investment.

[12] We use one asterisk for the buyer's first-best equilibrium, and two asterisks for the social optimum.

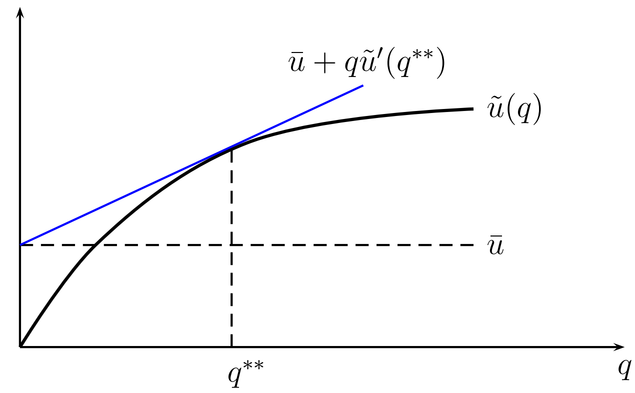

Figure 1: Determination of socially optimal supply

3.2 Buyer's first-best

Consider then what would be the first-best for the buyer side. This corresponds to a situation where producers are perfectly competitive, and the time of investment is chosen to maximize $W(s_t)$ only. Competitive sellers rationally foresee when the buyer side is going to invest, and based on this, they choose a constant supply path to equalize prices across times before and after the investment. We can copy the template from the social optimum to show that along the consumers' first-best path, welfare $W(.)$ is linear, i.e., $W(s) = \lambda s$ for some constant λ.

In figure 1, we can maximize the buyer's value of the resource if we find the supply level q^* on the curve of utility surplus $u(q)$ where the line through $(0, \overline{u})$ and $(q, u(q))$ has the steepest slope. The solution with $T > 0$ either takes the maximum demand level, with optimal supply $q^* = D(0)$, or otherwise, optimal supply q^* must satisfy

$$u(q^*) = \overline{u} + q^* u'(q^*). \tag{11}$$

We have a simple graphical determination of the consumers' optimum, which is generically unique as $u(.)$ is nonlinear everywhere.[13] In turn q^* determines the date of investment, by $T + k = s_0/q^*$. Relative to the social optimum, consumers can increase their payoff by forcing the seller to sell the resource faster:

[13]Graphically, the determination of q^* is analogous to the determination of q^{**} in Figure 1. The difference is that $u(.)$ should substitute for $\tilde{u}(.)$, and that $u(.)$ need not be concave. The solution is generically unique since a slight perturbation of parameters, e.g., $\bar{u}$ would eliminate a possible case of multiplicity.

Proposition 2 *The resource supply in the buyers' optimum exceeds resource supply in the social optimum: $q^* > q^{**}$. The resulting time interval of resource dependence is shorter than in the social optimum. The supply q^* increases and the resource dependence time interval decreases with the level of the substitute utility.*

Proof. From (11) and $u' > 0$, it follows that $u(q^*) > \overline{u}$, and thus $q^* > q^{**}$. The second part is straightforward. Consider two substitute utility levels $\overline{u}_A$ and $\overline{u}_B$ with associated optimum q_A^* and q_B^*, respectively. As the optimum maximizes the excursion of the utility surplus, we have

$$\begin{aligned} \frac{u(q_A^*) - \overline{u}_A}{q_A^*} &\geq \frac{u(q_B^*) - \overline{u}_A}{q_B^*} \\ \frac{u(q_A^*) - \overline{u}_B}{q_A^*} &\leq \frac{u(q_B^*) - \overline{u}_B}{q_B^*} \end{aligned}$$

Rearranging gives

$$(\overline{u}_A - \overline{u}_B)(q_A^* - q_B^*) \geq 0.$$

■

The opposing interests are now clear: the seller would like to delay investment as much as possible (to spread supplies thinly over time as flow profits are concave), the social optimum requires that consumers at least receive reservation utility, and the buyer prefers even faster depletion.[14] It is obvious that in the equilibrium of the game supplies and investment time must lie between the extremes identified here. Furthermore, a better substitute increases supplies both in first-best and in the buyer's optimum.

For the analysis of the strategic interaction, the following assumptions will simplify the exposition. Let $q^m = \arg\max \pi(q)$ and define $Z = [0, q^m]$. The seller will never supply more than q^m as flow profits decrease with higher supplies. We assume that both the initial stock and the maximum supply level q^m are sufficiently large:

$$u(q) - qu'(q) > \overline{u} \text{ for } q = s_0/k \text{ and } q = q^m. \tag{12}$$

Because of continuity and violation of the inequality for $q = 0$, assumption (12) implies $0 < q^* < q^m$ and $q^* < s_0/k$, and thus it rules out immediate investment. It also implies concavity of $u(.)$ around q^*, and ensures that the buyer will be satisfied with less than the

[14]These results are consistent with the common view that the seller's market power makes the resource-depletion path more conservative (see Hotelling 1931). Buyers' market power speeds up consumption both in the optimal tariff literature (see Karp-Newbery 1993) and strategic R&D and technology literature (see the papers cited in the introduction).

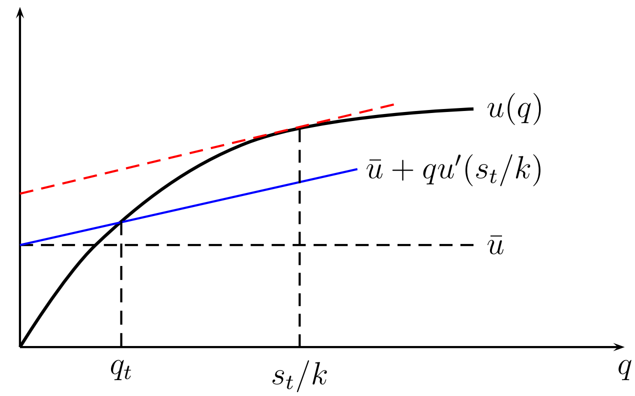

Figure 2: Determination of equilibrium q_t

seller's maximum conceivable supply q^m. Recall that a larger q^* follows from a greater long-run surplus $\bar{u}$: the buyer wants to consume the resource faster the better is the outside option.

Definition 1 *The buyer has a weak substitute if $q^* < q^m$. Otherwise, the substitute is strong.*

Assumption (12) implies that the substitute is weak.[15] Note that this is not only an assumption on the outside option $\bar{u}$ but also on preferences; it rules out, e.g., globally convex surplus function $u(q)$, which would violate (12).

For future reference, we define the buyer's first-best marginal value of the resource as

$$\lambda^* = [u(q^*) - \overline{u}]/q^*. \tag{13}$$

In the buyers' optimum, the consumer share of total resource surplus $V(s_0) + W(s_0)$ is $\lambda^* s_0$, and the seller receives the remainder.

3.3 First look at equilibrium: investment indifference

As we will show formally in Section 4, the key to the characterization of the equilibrium is the seller's strategy to keep the buyer side indifferent between the following two actions: (i) invest today and consume the remaining stock during the transition time interval

[15]For the analysis of the strong substitute cases that we do not consider in this paper, we refer to Gerlagh and Liski (2007).

k, and (ii) postpone the decision by one unit of time, maintaining the possibility for investing tomorrow. The seller postpones investment as long as possible by sustaining the buyer's indifference. When time is continuous, the indifference can be characterized, at each time t, by

$$u(q_t) = \bar{u} + q_t u'(s_t/k). \tag{14}$$

Under the postulated indifference, surplus $u(q_t)$ should cover *the cost from postponing* the long-run surplus flow $\bar{u}$ by a unit of time, and *the cost from depleting* the stock at rate q_t.[16] In view of Fig. 2, which depicts a concave surplus frontier and a line summing up the two cost terms for a given s_t, we see that the supply making the indifference to hold is defined by the intersection of the surplus curve (left-hand side of (14) as a function of q_t) and the cost curve (right-hand side for given s_t). As the resource is depleted, s_t/k declines. When the stock s_t approaches kq^* from above, the slope of the cost curve increases and, therefore, quantity q_t needed for the indifference must increase as well:[17]

$$\frac{dq_t}{ds_t} = \frac{q_t u''(s_t/k)}{k(u'(q_t) - u'(s_t/k))} < 0 \text{ for } q_t < s_t/k, \tag{15}$$

as the numerator is negative while the denominator is positive. Thus, to postpone the investment, *supplies must increase when the remaining resource stock declines*, until the point where the buyer's optimum given by (11) and the indifference (14) coincide. That is, the buyer will always invest when by doing so the buyer's first-best can be implemented. The resource level at which investment must take place s^*, is thus defined by the buyer's first-best supply q^*,

$$s^* = kq^*.$$

It follows that at the time of investment, supplies under continuation and after investment coincide, at level q^*. When the stock s_t approaches s^*, the overall path of supplies increases up to the point of investment, after which it is constant.

[16]We immediately see that this condition closely resembles the buyer's optimum (11). There is one important distinction. While the right-hand-side of the buyer's optimum indifference condition (11) includes the constant marginal value of the initial resource and so defines a constant q^*, the strategic buyer's indifference condition (18) is based on the marginal value of the current resource stock and so it defines a supply scheme q_t that is dependent on the current resource level s_t.

[17]Note that the conclusion holds even without the global concavity of $u(.)$: it is an implication of assumption (12) that $u(.)$ is locally concave in the neighborhood of q^*.

4 Strategic resource dependence

There are $i = 1, ..., N$ intervals of time (also referred to as periods). The strategic interaction between the buyer and the seller can take place at the beginning of each interval. Each discrete period lasts for a time interval of lenght ε, and actions cannot be altered during each such interval. Thus, time t runs continuously but agents can make decisions only at time points $t_i = \varepsilon(i-1)$. At each t_i, the buyer has a binary choice variable, $d_{t_i} \in \{0, 1\}$, where $d_{t_i} = 1$ indicates that the buyer adopts the substitute at t_i. Substitute adoption means that the buyer announces the termination of the relationship with the seller. In the game, this means that stopping payoffs will be realized. At each t_i, the seller's only choice variable is supply $q_{t_i} \in \mathbb{R}_+$.

At each t_i, there are three stages:

1. the seller chooses q_{t_i};
2. the buyer chooses $d_{t_i} \in \{0, 1\}$;
3. market clears with q_{t_i} at each $t \in [t_i, t_i + \varepsilon)$ if $d_{t_i} = 0$, or, if $d_{t_i} = 1$, the strategic interaction stops and the stopping payoffs are realized.

For the buyer, the stopping payoff is

$$W^I(s) = k(\widehat{u}(s/k) - \bar{u}),$$

where $\widehat{u}(s/k) = u(\min\{s/k, q^m\})$, as seller types $s > kq^m$ will not supply above the level q^m. Note that $W^I(s_t)$ is defined as the value of the excursion above the long-run payoff, measured from the stopping time onwards. For the seller, the stopping payoff is

$$V^I(s) = k\hat{\pi}(s/k),$$

where $\hat{\pi}(s/k) = \pi(\min\{s/k, q^m\})$.

In Appendix 7, we show that there is a unique subgame-perfect equilibrium in this stopping game for each finite N. We also show that the equilibrium is stationary in the sense that the strategies become independent of N for sufficient large N. Finally, we show that uniqueness and stationarity properties are sustained when ε becomes arbitrarily small, and N arbitrarily large.[18] In the main text below, we construct the equilibrium for the case where ε is arbitrarily small, using differential methods.

[18] The appendix shows how the limit is taken.

We can thus look for Markov-perfect strategies for the continuation regime C (all strategic interaction takes place in this state). For the seller, the strategy is a function

$$\eta : \mathbb{R}_+ \mapsto Z,$$

where $q_t = \eta(s_t) \in Z = [0, q^m]$ is the seller's supply offered to the market at t. The buyer observes q_t and makes a stopping decision

$$\mu : \mathbb{R}_+ \times Z \mapsto \{0, 1\},$$

where $d_t = \mu(s_t, q_t) = 1$ implies that the stopping payoffs are realized at t.[19]

4.1 The buyer's problem

The seller has a strategy $q_t = \eta(s_t)$, and based on the seller's strategy we find the strategy for the buyer to invest. We characterize the strategy for an arbitrarily small ε. Assume now that $\eta(s_t)$ is constant over a short interval of time $[t, t+\varepsilon]$, and write the expression for the payoff before the investment as

$$W(s_t) = \max_{d_t \in \{0,1\}} \{[\varepsilon u(\eta(s_t)) - \varepsilon \bar{u} + W(s_t - \varepsilon \eta(s_t))](1 - d_t) + W^I(s_t) d_t\}. \qquad (16)$$

The term $\varepsilon \bar{u}$ is the direct cost from postponing the investment since the buyer side loses the long-run surplus $\bar{u}$ for ε interval of time by not investing at t. As ε approaches zero, (16) can be approximated as follows:

$$W(s_t) = \max_{d_t \in \{0,1\}} \{[\varepsilon u_t - \varepsilon \bar{u} - \varepsilon q_t W'(s_t) + W(s_t)](1 - d) + W^I(s_t) d\}, \qquad (17)$$

where we use shorthands $u_t = u(\eta(s_t))$ and $q_t = \eta(s_t)$. Thus, we must have $W(s_t) \geq W^I(s_t)$. If a strict inequality applies, $W(s_t) > W^I(s_t)$ and $\mu(s_t, q_t) = 0$, for all $q_t \geq 0$. That is, if the buyer's payoff before investment strictly exceeds the post-investment payoff, then (for sufficiently small ε) the buyer will not invest irrespective of the offer.

Thus, if choosing $d = 0$ is optimal, then $W(s_t) \geq W^I(s_t)$ and

$$u_t = \bar{u} + q_t W'(s_t). \qquad (18)$$

This is the key condition throughout this paper. It says that the consumer surplus under continuation of the resource dependence, u_t, covers the direct cost from continuing, $\bar{u}$, and

[19] Note that because of the timing assumption (the three stages above), the buyer's Markov strategy depends not only on the state but also on the seller's offer. In this respect, a similar formulation is used in Felli and Harris (1996) and Bergemann and Välimäki (1996). In Appendix 8 we explain the role of the timing assumptions.

the marginal reduction in payoff from the fact that the stock available for consumption during the remaining overall time interval of resource dependence is depleted, $q_t W'(s_t)$. The relation also ensures that $W(.)$ and $W'(s_t)$ are continuous almost everywhere if $q_t = \eta(s_t)$ is continuous almost everywhere.

4.2 The seller's problem

For the seller's problem we follow the same approximation procedure as above. For short time interval ε, and given the buyer's strategy $d_t = \mu(s_t, q_t)$, supply in the next ε interval of time is q_t if $\mu(s_t, q_t) = 0$. If $\mu(s_t, q_t) = 1$, the stopping payoffs are realized. The seller's best response satisfies

$$V(s_t) = \max_{\{q_t\}} \{[\varepsilon\pi(q_t) + V(s_t - \varepsilon q_t)](1 - \mu(s_t, q_t)) + V^I(s_t)\mu(s_t, q_t)\}. \tag{19}$$

When ε approaches zero, this value can be approximated by (letting $\mu(\cdot) = \mu(s_t, q_t)$):

$$V(s_t) = \max_{\{q_t\}} \{[\varepsilon\pi(q_t) - \varepsilon q_t V'(s_t) + V(s_t)](1 - \mu(\cdot)) + V^I(s_t)\mu(\cdot)\}. \tag{20}$$

Given $\mu(s_t, q_t)$, the seller can choose if there will be investment or not. If choice $\mu = 0$ is implemented, then by (20), we must have

$$-q_t V'(s_t) + \pi(q_t) = 0. \tag{21}$$

If choice $\mu = 1$ is implemented, then

$$V(s_t) = V^I(s_t). \tag{22}$$

From these conditions we can immediately see that the seller always prefers to continue the relationship irrespective of the stock level. Recall that s^* denotes the stock level at which the buyer's first-best is to invest.

Lemma 1 *If $s_0 \geqslant s^*$ and for $s_t > s^*$ some path q_t exists, continuous almost everywhere, with $q_t \leq s_t/k$ and $\mu(s_t, q_t) = 0$, then the seller prefers continuation to stopping. In particular, $V(s^*) = V^I(s^*)$, $V'(s_t) > V^{I\prime}(s_t)$ for all $s_t \geqslant s^*$, and thus $V(s_t) > V^I(s_t)$.*

Proof. Equality at s^* follows from the fact that the buyer's first-best is to invests at s^*, and thus seller's continuation payoff is independent of the offer. Thus, $V(s^*) = V^I(s^*)$. Assuming $q_t \leq s_t/k$, we have

$$V'(s_t) = \psi(q_t) \geq \psi(\frac{s_t}{k}) > \psi(\frac{s_t}{k}) + \frac{s_t}{k}\psi'(\frac{s_t}{k}) \geq V^{I\prime}(s_t).$$

The first equality follows from (21), the second (weak) inequality is by assumption ($q_t \leq s_t/k$), the third (strict) inequality follows from a negative price slope, and the last (weak) inequality follows from the definition of $V^I(s_t)$. By integration, $V(s_t) > V^I(s_t)$ follows. ■

Thus, the 'smooth pasting' condition does not hold for the seller for an intuitively obvious reason: the buyer's decision to invest implies a binding time-to-sell constraint for the seller. The seller will not end the dependence before the buyer wants to end it, as it would be profitable to spread sales as thinly as possible over time.[20] For this reason, when the stock level is public knowledge and $q_t \leq s_t/k$ is acceptable to the buyer, it will be the buyer's indifference that determines the time to end the resource dependency.

4.3 Equilibrium

Establishing and characterizing the equilibrium supply is a simple undertaking based on the analysis of buyer's indifference between continuation and stopping, given that the seller never prefers stopping. We first prove that $W^I(s)$ defines the buyer's welfare any time before investment.

Lemma 2 *In equilibrium, the buyer is indifferent between continuing the resource dependence and investing at any given t prior to the investment date:*

$$W(s_t) = W^I(s_t) \text{ for all } s_t \geq s^* \tag{23}$$

Proof. The proof is by contradiction. Assume $W(s) > W^I(s)$ at some $s > s^*$. Note that (i) function $W^I(.)$ is continuous, (ii) function $W(.)$ is continuous by (2), and (iii) the slope of $W(.)$ is bounded from above by λ^* defined in (13). That is, for all δ

$$W(s - \delta) \geq W(s) - \delta\lambda^*. \tag{24}$$

Now, if we take $\hat{\delta} = \delta$ sufficiently small such that $\hat{\delta}\lambda^* < W(s) - W^I(s)$, we have for all $s_\tau \in [s - \hat{\delta}, s]$

$$W(s_\tau) \geq W(s) - (s - s_\tau)\lambda^* \geq W(s) - \hat{\delta}\lambda^* > W^I(s) > W^I(s_\tau) \tag{25}$$

where the first inequality follows from (24), the second inequality follows from $s - s_\tau \leq \hat{\delta}$, the third inequality from the definition of $\hat{\delta}$, and the last inequality follows from the fact

[20] We will derive this same condition also with discounting but there we need restrictions on the utility formulation.

that $W^I(.)$ is increasing. The inequality $W(s_\tau) > W^I(s_\tau)$ implies that the buyer's best response is $\mu(s_\tau, q_\tau) = 0$ for all $q_\tau \geq 0$ and $s_\tau \in [s - \hat{\delta}, s]$. The seller is not constrained to reduce supplies. By (21), $V'(s)$ increases in the price, i.e., as q declines. The seller can now decrease supplies and so extend the time interval of resource dependence to infinity. Suppose the seller chooses some supply q_τ for time interval $\tau \in [t, t + \Delta]$, with $\int_t^{t+\Delta} q_\tau d\tau = \hat{\delta}$. The seller's payoff amounts to $V(s_t) = \int_t^{t+\Delta} \pi(q_\tau)dt + V(s_t - \hat{\delta})$. Alternatively, the seller could spread the same supply over double time, $\tilde{q}_\tau = \frac{1}{2}q_{t+\frac{1}{2}(\tau - t)}$, $\int_t^{t+2\Delta} \tilde{q}_\tau d\tau = \hat{\delta}$. The seller's profit would increase by $\int_t^{t+\Delta} 2\pi(\frac{1}{2}q_\tau) - \pi(q_\tau)d\tau > 0$. Since profits are strictly concave, the payoff is maximized by letting, $q_\tau \to 0$ and $\Delta \to \infty$. But if supplies drop close to zero, for a time interval of unbounded length, then the buyer's utility excursion from the long-run payoff $\bar{u}$ becomes negative:

$$W(s_t) = \int_t^{t+\Delta}[u(q_\tau) - \overline{u}]dt + W(s_t - \hat{\delta}) < 0$$

as $q_\tau \to 0$ and $\Delta \to \infty$. This contradicts $W(s_t) > W^I(s_t)$. ■

It is thus the buyer's indifference that determines equilibrium supply policy, $q_t = \eta(s_t)$. The buyer's indifference condition (23) together with (18) requires

$$u(q_t) = \bar{u} + q_t u'(s_t/k) \text{ if } s_t < kq^m \quad (26)$$

$$u(q_t) = \bar{u} \text{ otherwise.} \quad (27)$$

This is a slightly adjusted version of (14) because $W_t'(s_t) = u'(s_t/k)$ when $s_t < kq^m$, but $W_t'(s_t) = 0$ otherwise as the stock level does not affect supply post-investment if $s_t > kq^m$. We have already illustrated this indifference for a concave surplus in Fig. 2. Recall that the investment point satisfies $q_t = s^*/k = q^*$, which is the buyer's first-best supply as it maximizes the buyer's payoff from this stock level onwards. The seller cannot compensate the buyer for continuation after the stock has fallen just below s^* because the buyer can implement his first-best by ending the relationship there. The scarcity cost exceeds the maximal marginal value of the resource,

$$W'(s_t) > \lambda^* = [u(q^*) - \overline{u}]/q^*,$$

when $s_t < s^*$ because u is locally concave around q^*.

We now describe the general case where u is not necessarily globally concave. Recall that by assumption (12), continuation can provide some surplus over investment at $t = 0$: there is some $q_0 < q^* < q^m$ such that

$$u(q_0) = \bar{u} + q_0 u'(s_0/k).$$

The seller can thus entice the buyer to continue by supplying $q_0 < s_0/k$ such that the buyer's indifference as described in (26)-(27) holds. At time T of investment, the indifference condition will be

$$u(q^*) = \bar{u} + q^* u'(s_T). \tag{28}$$

If the resource level satisfies $s_T = s^*$, then the buyer will invest at this point as the buyer's first-best can be implemented. However, since the consumer surplus is not generally concave, equation (28) may be satisfied at some larger resource level, $s_T > s^*$, but not for a stock marginally smaller than s_T. Then, the buyer stops the game at s_T. The continuation region in the stock space is thus defined by the lowest resource stock level such that the indifference conditions can be met over the entire interval $[s_T, s_0]$:

$$s_T = \min\{s | u'(s') \leq \lambda^* \text{ for all } s' \in [s, s_0]\}$$

We have by construction $u'(s_t/k) \leq u'(q^*)$ for all $s_T \leq s_t \leq s_0$. By continuity of $u(.)$, supply $q_t = \eta(s_t)$ satisfying (26) to keep the buyer indifferent between stopping and continuing exists, and it varies with the remaining stock for $s_T < s_t < kq^m$.

Proposition 3 *For a given s_0, there exists a Markov-perfect equilibrium with s_T as defined above, q_t defined by (26)-(27), and investment taking place at s_T, $q_T = q^*$.*

Proof. From above it is clear that we have described the buyer's best-response to the supply policy. It remains to be shown that the seller cannot deviate from the supply. For $s_t \geqslant s_T$ we have $q_t \leq s_t/k$ along the equilibrium path, and thus $V(s_t) > V^I(s_t)$ by Lemma 1. The seller thus prefers to continue at each point along the equilibrium path. Moreover, since $V'(s_t) = p_t$, the lowest feasible supply maximizes the seller's payoff, i.e., the indifference-making supply for the buyer. ■

Under nonconcave surplus, the increase in supply over time may not be monotonic as the buyer's scarcity cost $u'(s_t/k)$ may not be monotonic (u'' may change sign). However, when the equilibrium path approaches the investment point, supplies must increase, so that our main conclusion holds irrespective of the functional forms.

Proposition 4 *The equilibrium supply path q_t is*

1. *constant at level $u^{-1}(\bar{u})$ when $s_t > kq^m$;*

2. *varying over time in $u^{-1}(\bar{u}) \leq q_t \leq q^*$ when $s_T < s_t < kq^m$, but ultimately increasing to q^* as s_t approaches s_T;*

3. *strictly increasing for all $s_T = s^* < s_t < kq^m$ if consumer surplus $u(.)$ is concave*

4.4 Discussion

The assumption that it takes time to change the demand is necessary for our result that supplies increase over time. When the time-to-build delay k is extremely short, the buyer knows that the alternative surplus flow $\bar{u}$ will arrive almost immediately after investment. Then, the buyer's outside option is just the long-run surplus, and the seller needs to supply only $u^{-1}(\bar{u})$ to keep him indifferent. The seller will receive the whole surplus and, therefore, he will implement the first best.

A larger k captures the idea of having capacity constraints in making a fast transition to the substitute. The buyer will feel the scarcity cost from a decreasing stock for a longer period and, therefore, will require a larger compensation to continue without investment. A larger k thus means that the buyer will realize earlier that there is scarcity during the transition period, and the upward pressure on supplies will start earlier, i.e., at higher current stock levels. In this sense, the buyer's outside option is more sensitive to the stock level s_t, and he will be able capture larger part of the overall surplus.

The above simple formalization of the time-to-build delay captures quite well a general idea. Let us now briefly discuss alternative but qualitatively equivalent ways of formalizing the transition to the substitute. First, the buyer's decision could trigger a gradual adjustment of the demand rather than the above one-time event taking place after the time-to-build period. One way to formalize a gradual adjustment is to assume an exogenous rate of decline for the fraction of the demand still depending on the resource. This would change essentially nothing in our main model. Another way to proceed is to assume that at each period after making the decision $d = 1$, the buyer chooses an investment rate, i.e., how many units of demand to switch. If investment cost is linear, the buyer can switch all units at once, which would lead to an equilibrium equivalent to the one obtained when k is almost zero in our main model. When adjustment (investment) costs are strictly convex, possibly with a per period capacity constraint on the rate of change, then the buyer cannot change his dependence on the resource quickly, and the equilibrium dynamics come close to those achieved under $k > 0$ in our model. In this sense, k captures adjustment costs in the demand change.

Second, uncertainty regarding the transition to the substitute can be captured in many ways. A simple extension is to treat k as a random variable, which would not affect the nature of our results in a material way. Alternatively, the buyer's decision $d = 1$ could trigger a random process with a downward trend for the fraction of demand still depending on the resource. The seller would face stochastic demand over stochastic time horizon but the ex ante values from entering this phase could still be evaluated in a straightforward way for both players, and the strategic interaction before investment would not essentially differ from what we have described.

Let us then finally discuss our assumption that the resource cannot compete with the substitute, once in place. Recall that we abstract from the substitute's marginal production costs and resource extraction costs. We could as well have assumed that marginal production costs for the substitute fall short of resource extraction costs so that the resource has no use when the substitute is in place. In Appendix 9 we explain how the competition between the substitute and resource in the long-run state can be included in the model. Essentially, the long-run competition reduces the stock available for strategic interaction in the continuation state. The main features of the equilibrium are not altered by this inclusion.

5 Discounting

Discounting is an important element in resource use. In the traditional Hotelling model, discounting is what distinguishes markets at different dates, which, in the presence of seller power leads to intertemporally differentiated prices. Another reason for such differentiation is the buyer's changing opportunity cost of continued the resource dependence, and this effect we have identified in the undiscounted analysis. Two main questions remain. First, does the discounted equilibrium converge to the undiscounted limit we have described? Second, does the undiscounted equilibrium describe well the essence of the discounted case. To answer the latter question, we consider the discounted equilibrium for a particular utility function (CRRA).

For the first question, note that our model assumes a payoff criterion that corresponds to Dutta's (1991) strong long-run average criterion, where the per-period payoffs are net of the long-run average values. In our case, the long-run average values are trivially defined by $\bar{u}$ for the buyer and by 0 for the seller, so the concept is well-defined. Starting from different initial states cannot lead to arbitrarily large value differences in our model since the long-run payoffs for the programs are equivalent and the transitory payoffs are bounded. The Dutta's conditions for value boundedness and finiteness are satisfied and, therefore, the discounted values and policies converge to those given by the strong long-run average criterion in the undiscounted case.

Let us now consider whether sufficiently large positive discounting can add features to the equilibrium characterization. Assume that the continuous-time discount rate is positive, $r > 0$. Let $V^I(s)$ denote the seller's discounted payoff at the time of buyer's stopping decision. For the seller, there is a well-defined unique monopoly supply path, equalizing present-value marginal revenues over the remaining time interval, leading to the value $V^I(s)$. In the continuation state, the seller's optimal sale q_t is a best-response to stopping rule $\mu(s_t, q_t)$ satisfying

$$V(s_t) = \max_{\{q_t\}}\{[\varepsilon\pi(q_t) + e^{-\epsilon r}V(s_t - \varepsilon q_t)](1 - \mu(s_t, q_t)) + V^I(s_t)\mu(s_t, q_t)\}. \qquad (29)$$

As in the undiscounted equation (19), the strategies are defined over some arbitrarily small interval of time ε, so that the continuation value $V(s_t)$ satisfies

$$-q_t V'(s_t) + \pi(q_t) - rV(s_t) = 0. \qquad (30)$$

Let us denote the true discounted payoff for the buyer by $U(s) = W(s) + \bar{u}/r$ in the continuation state, and by $U^I(s) = W^I(s) + \bar{u}/r$ at the time of stopping. Since the seller's supply path after stopping is unique, the payoff $U^I(s)$ is well defined. Given the seller's strategy $\eta(s_t)$, the buyer's continuation payoff $U(s_t)$ is given by

$$U(s_t) = \max_{d_t \in \{0,1\}}\{[\varepsilon u(\eta(s_t)) + e^{-\varepsilon r}U(s_t - \varepsilon\eta(s_t))](1 - d_t) + U^I(s_t)d_t\}.$$

For ε arbitrarily small, we find the positive discounting equivalent of (18):

$$u_t = rU(s_t) + q_t U'(s_t). \qquad (31)$$

When the buyer is indifferent between continuation and stopping, (31) holds with $U(s) = U^I(s)$. The interpretation then is that in addition to the depletion effect $qU^{I\prime}(s)$, the buyer must receive a return on the investment option, $rU^I(s)$. Replacing $U(s) = W(s) + \bar{u}/r$ and $U^I(s) = W^I(s) + \bar{u}/r$, and letting $r \to 0$ gives the undiscounted version of the optimality condition.

We can solve the equilibrium explicitly by assuming constant elasticity of demand $\epsilon = -\frac{1}{1-\sigma}$, associated with utility function, $\widetilde{u}(q) = q^\sigma$, where $0 < \sigma < 1$. Thus, $\psi(q) = \sigma q^{\sigma-1}$, $\pi(q) = \sigma q^\sigma$, and $u(q) = (1-\sigma)q^\sigma$. Under positive discounting, the supply q_t after stopping satisfies $\pi'(q_t) = e^{r(t-T+k)}\lambda$, for some $\lambda > 0$ (marginal revenues are equalized in present value). Using this condition, some manipulation gives

$$\begin{aligned} V^I(s) &= \sigma A s^\sigma && (32) \\ W^I(s) &= (1-\sigma)As^\sigma - \frac{1-e^{-rk}}{r}\bar{u}, && (33) \end{aligned}$$

where $A = \left(\frac{\omega}{1-e^{-\omega k}}\right)^{\sigma}\left(\frac{1-e^{-\omega\sigma k}}{\omega\sigma}\right)$ and $\omega = \frac{r}{1-\sigma}$. The buyer's indifference condition (31) becomes

$$q^{\sigma} = \frac{e^{-rk}\bar{u}}{1-\sigma} + rAs^{\sigma} + q\sigma As^{\sigma-1}. \tag{34}$$

Notice that when $r \to 0$, $A \to k^{1-\sigma}$, and we obtain

$$\begin{aligned} W^{I}(s) &= k[(1-\sigma)(s/k)^{\sigma} - \bar{u}] \\ V^{I}(s) &= k\sigma(s/k)^{\sigma} \\ (1-\sigma)q^{\sigma} &= \bar{u} + q\sigma(1-\sigma)(s/k)^{\sigma-1}, \end{aligned}$$

consistent with our ealier definitions for undiscounted $W^{I}(s)$, $V^{I}(s)$, and condition (26).

In the appendix, we show that supply is continuous at the investment point. This finding is used to prove that the seller prefers continuation to stopping at the time of investment, which ensures that (34) close to the point where stopping takes place. We can then use continuity of supply and (34) to establish the values for the resource stock and supply level at the investment point. Given σ, assume that k and r satisfy

$$\sigma(1-e^{-\omega k})^{\sigma} > 1 - e^{-\omega\sigma k}. \tag{35}$$

Under this condition we have (see the appendix)

$$s^{*} = \left[\frac{e^{-rk}\bar{u}}{(1-\sigma)^{2}A^{\frac{-\sigma}{1-\sigma}} - (1-\sigma)rA}\right]^{-1/\sigma} \tag{36}$$

$$q^{*} = A^{\frac{1}{\sigma-1}}s^{*} \tag{37}$$

We notice that (35) is equivalent to demanding that the denominator in (36) is positive. The condition holds for small rk, and is violated for large rk. When the condition (35) does not hold, the buyer will invest immediately. The value of rk determines the utility weight given to the long-term payoff. A lower weight for the long term implies that the buyer cares less about conserving the resource, and thus more easily invests, forcing the seller to exhaust the resource within the coming k time interval.

These findings lead to the following:

Proposition 5 *For constant elasticity of demand and (35) satisfied, equilibrium supplies first decline and then increase over time when s_0 is sufficiently large.*

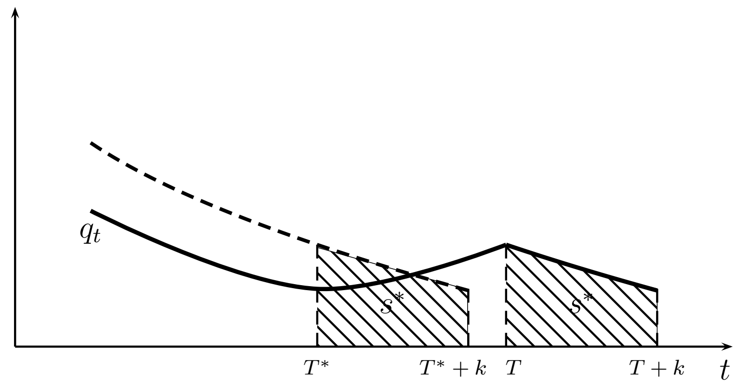

Figure 3: Equilibrium supply path under discounting

Proof. See Appendix. ■

We depict the equilibrium time path for supply in Fig. 3, as well as the buyer's optimal path. The latter involves choosing the highest supply path such that (i) prices are equal in present value, and (ii) the stock remaining at the investment time, T^*, is consumed during the technology transition time interval. The equilibrium value s^* is, like in the undiscounted case, exactly equal to the buyer's optimal s^* because, due to constant elasticity of demand, in the post-investment phase the seller supplies a competitive path in both cases: the constant demand elasticity eliminates the possibility of price discrimination at different dates after the investment (see Gilbert 1978). The two paths in Fig. 3 are therefore identical during the technology transition time interval, starting at T^* and T, respectively. However, before investment, the strategic seller can discriminate buyers at different dates according to (31) (the explicit constant elasticity of demand solution is given in (34)) and delay the arrival of the substitute as in the undiscounted case. When the stock is still large, supplies decrease over time as in the standard Hotelling model. When the stock becomes smaller and approaches s^*, supplies increase over time as in the undiscounted case because the buyer's indifference becomes binding.

6 Concluding remarks

In this paper, we considered strategic interactions between the seller of a depletable resource and consumers who have interests in ending their dependence on the resource. We modeled the situation using a framework that departs from explicit bargaining but still allows offers and responses. The approach seems relevant since there is significant coordination of actions on both sides of the oil market, for example, but at the same time explicit cooperation of the two sides is not feasible by the difficulty of enforcing international agreements. The key question in the relationship is when to start the process ending the resource dependence, that is, when to change the demand. The process changing the demand takes time and therefore a potentially significant fraction of the resource has to be saved for the transition time interval. Our insights to the problem follow from this simple allocation problem.

The main insight from our analysis is that producers' market power is reduced over time as continuing the relationship becomes more costly to consumers when the stock available for the demand transition is depleted. This means that changing the demand infrastructure becomes more relevant as a choice, leading to the conclusion that producers must increase supplies over time to postpone the buyer's action. In contrast with previous approaches to such strategic dependence, the basic implications of the Hotelling (1931) model are reversed.

What are the main lessons from these results for understanding the oil market? We believe it is the insight that energy technology policies in oil-importing countries can act as an increasingly effective strategic instrument, in part destroying producers scarcity rents. While in general this insight is not new, our approach is new as it accounts for the fact that the transition is not an immediate event, and this insight results in explicitly increasing supplies in a stationary market environment.

There are several well-established explanations why scarcity rents do not seem to drive supply behavior in oil or other exhaustible resource markets: declining extraction costs due to technological progress can lead to U-shaped price paths; durability of the final good; learning of new reserves; and imperfect competition (see Gaudet 2007) for a review of the literature). Our explanation is complementary and distinct from previous explanations presented in this literature.

On a theoretical level, there are some obvious extensions. As we have seen, the size of the remaining stock is what determines the seller's ability to entice the buyer side to postpone actions ending the resource dependence: it is critical for the buyer to observe how much resource is left for the transition, otherwise the seller can take advantage of the buyer's imperfect information for the right timing of the demand change. Recall that the larger is the stock, the lower is the equilibrium supply (at earlier points on the sales path, stocks are larger). In this precise sense, a large stock implies more power to reduce supplies than a small stock. If the stock is not observed by the buyer side, a small seller can potentially mimic large seller's policy of reducing supplies and, thereby, extend the investment date from what would otherwise hold for the small seller.

The above observation suggests an extension to situations where there is asymmetric information about the size of the seller's resource stock. The study of asymmetric information in resource extraction can also be motivated by the developments in the oil market. The core reserves of cheap oil are not managed like most productive assets in market economies; management of cheap oil is characterized by secrecy. The dynasties of the Middle East do not disclose technical production information and make efforts to prevent auditing of the reserves. The future availability of conventional oil is a major public concern in oil importing countries; industry experts' opinions on the size of economically viable stocks diverge widely.[21] We have presented a preliminary analysis of the asymmetric information equilibrium in our working paper Gerlagh-Liski (2007).[22]

Other extensions are the following. Adding a fringe of competitive producers would reduce the seller's market power in a rather straightforward way; the fringe would free-ride on the seller's market power by selling first when the prices are high. Uncertainty about the technology transition time interval would affect the precise timing of investment and the level of the supply path, but not the basic insights. A less straightforward extension is a reversed asymmetric information situation where the buyer side privately knows whether the adoption decision has been made but the resource stock size is public information. Alternatively, under the R&D interpretation, the buyer privately knows the state of the technology. We leave these interesting topics open for future research.

7 Appendix: Discrete time model

We consider a discrete-time model of length N and show the following properties:

1. The subgame-perfect equilibrium in the discrete-time game is unique (Lemma 3);

2. The stopping time has bound M^* which depends on the period length ε (Lemma 4);

[21]These concerns are reviewed in the Hirsch report. A book by Matthew R. Simmons (2005) explicates carefully the industry experts concerns regarding the Saudi stocks. While it is hard to judge the validity of the arguments in general, one cannot escape the fact that the market cannot evaluate the maturity of the main Saudi oil fields; Saudi Aramco has not disclosed technical production information since the early 1980s (Simmons).

[22]Saure (2008) also considers resource extraction under asymmetric information in a two-period model.

3. The equilibrium strategies are stationary for sufficiently large N (Lemma 5).

The required length N for stationarity is defined in terms of the bound M^* which in turns depends on the period length ε. This will allow us to take ε arbitrarily close to zero and N arbitrarily large, while maintaining properties 1 and 3.

Consider now the game with $i = 1, ..., N$ intervals of time. Each discrete period lasts for a time interval of length ε, and actions cannot be altered during each interval. Thus, time t runs continuously but agents can make decisions only at time points $t_i = \varepsilon(i-1)$. At each t_i, there are three sub-stages: (i) the seller chooses q_{t_i}; (ii) the buyer chooses $d_{t_i} \in \{0,1\} \equiv I$; (iii) market clears with q_{t_i} at each $t \in [t_i, t_i+\varepsilon)$ if $d_{t_i} = 0$, or, if $d_{t_i} = 1$, the strategic interaction stops and the stopping-payoffs are realized. We denote the stopping period by $M+1$, so that M is the number of periods in which the offer is accepted, $M \in \{0, ..., N\}$, where $M = N$ if the buyer maintains $d_{t_i} = 0$ in all periods.

The final outcome of the game is (M, h_M) where

$$h_M = (q_{t_1}, q_{t_2}, ..., q_{t_M}) \in \mathbb{R}_+^M.$$

Similarly, we define the vector h_i as the relevant history at t_i, with null string h_1 and $h_i = (h_{i-1}, q_{t_{i-1}})$. The investment decision is not included in the history as the history only develops as long as no investment has taken place prior to i. The strategy for the seller is the collection of functions

$$Q = (Q_1(\cdot), Q_2(\cdot), ..., Q_N(\cdot)),$$

where

$$Q_i(h_i) : \mathbb{R}_+^{i-1} \longmapsto \mathbb{R}_+.$$

The strategy for the buyer is

$$D = (D_1(\cdot), D_2(\cdot), ..., D_N(\cdot))$$

where the strategy depends on both history and the offer

$$D_i(h_i, q_{t_i}) : \mathbb{R}_+^i \longmapsto I.$$

Given the history h_i, strategies (Q, D) generate an outcome (M, h_M) and the development of the stock $s_i = s_{t_i}$.The buyer's payoff at time t_i is

$$W_i(h_i, Q, D) = \sum\nolimits_{n=i}^{M} \varepsilon u(q_{t_n}) + W^I(s_{t_{M+1}}) \tag{38}$$

where

$$W^I(s_{t_{M+1}}) = k(u(\frac{s_{t_{M+1}}}{k}) - \bar{u}). \tag{39}$$

The seller's payoff is

$$V_t(h_i, Q, D) = \sum\nolimits_{n=i}^{M} \varepsilon\pi(q_{t_n}) + V^I(s_{t_{M+1}}) \tag{40}$$

where

$$V^I(s_{t_{M+1}}) = k\pi(\frac{s_{t_{M+1}}}{k}). \tag{41}$$

Definition 2 *Strategy (Q^*, D^*) is an equilibrium strategy if for any h_i:*

- *Q^* maximizes $V_i(h_i, Q, D^*)$*
- *D^* maximizes $W_i(h_i, Q^*, D)$.*

When the seller is indifferent between two strategies, we require that the seller chooses the smallest supply. Also, when the buyer is indifferent between stopping and continuation, we require the buyer to continue. Denote the game defined this way by $\Omega(s_0, N)$. We are interested in subgame perfect equilibria (SPE). The SPE have the advantage that they are constructed backwards which enables us to prove uniqueness and existence below. A consequence of this feature is that the strategy functions are conditioned on the number of offers that can be made. We count backwards from N. Thus, we use a subscript $n = N + 1 - i$, where i is the period. At the start of the first period, $i = 1$ and $n = N$. At the start of the second period, $i = 2$ and $n = N - 1$, and so forth. The lemma below shows that strategies are conditional on n rather than on i.

Lemma 3 *For $\Omega(s_0, N)$, there exists a unique SPE, which is a Markov equilibrium: the supply function and decision function only depend on current stocks, the current offer, and the number of periods to go before the game has to end, $n = N + 1 - i$. We switch subscripts from i to n. There exist functions $\eta_n : \mathbb{R}_+ \longmapsto \mathbb{R}_+$, $\mu_n : \mathbb{R}_+^2 \longmapsto I$, such that for all $n = 1, ..., N$*

$$\begin{aligned} Q_n^*(h_n) &= \eta_n(s_n) \\ D_n^*(h_n, q_n) &= \mu_n(s_n, q_n). \end{aligned}$$

Payoffs are also only dependent on current stocks and n

$$\begin{aligned} V_n^*(h_n) &= V_n(s_n) \\ W_n^*(h_n) &= W_n(s_n). \end{aligned}$$

Proof. We construct the equilibrium payoffs and best-responses backwards from $i = N$ to $i = 1$, that is, from $n = 1$ to $n = N$. We will use induction to prove the lemma. Consider $n = 1$; the seller can make one offer. For given stock s, the acceptance set is

$$\Gamma_1(s) = \{q > 0 | \varepsilon u(q) - \varepsilon\bar{u} + W^I(s - \varepsilon q) \geq W^I(s)\},$$

where we removed time subscripts for convenience. The buyer's best response is the indicator function such that $\mu_1(s, q) = 0$ iff $q \in \Gamma_1(s)$. Notice that this construction implies that the buyer continues dependence if indifferent. As profits are strictly concave, if $\Gamma_1(s) \neq \varnothing$, the seller will always choose an offer that is accepted. The seller's payoff at $n = 1$ is

$$V_1(s) = \begin{cases} V^I(s) \text{ if } \Gamma_1(s) = \varnothing \\ \max_q\{\varepsilon\pi(q) + V^I(s - \varepsilon q) | q \in \Gamma_1(s)\} \text{ otherwise.} \end{cases} \tag{42}$$

Above, we required that when the seller has identical payoff for different supply levels, the lowest is chosen. The payoff then defines unambiguously the seller's strategy $q_1 = \eta_1(s)$ for $\Gamma_1(s) \neq \varnothing$, and the payoffs $V_1(s)$ and $W_1(s) = \varepsilon u(\eta_1(s)) - \varepsilon\bar{u} + W^I(s - \varepsilon\eta_1(s))$. For $\Gamma_1(s) = \varnothing$, $W_1(s) = W^I(s)$, $W_1(s) = W^I(s)$ and the last-period seller's strategy is irrelevant. This proves the lemma for $n = 1$.

We will now use induction from n to $n+1$, assuming that the lemma holds for n. Let the maximum number of remaining offers be $n + 1$. The buyer knows that if it accepts the offer, it will enter the game with a maximum of n offers. The buyer accepts the offer iff it is in the set

$$\Gamma_{n+1}(s) = \{q > 0 | \varepsilon u(q) - \varepsilon\bar{u} + W_n(s - \varepsilon q) \geq W^I(s)\}, \tag{43}$$

where W_n is the equilibrium continuation payoff, and $W^I(s)$ is the stopping payoff. The buyer's best response is the indicator function such that $\mu_{n+1}(s, q) = 0$ iff $q \in \Gamma_{n+1}(s)$. Similarly, the sellers payoff at $n + 1$ is

$$V_{n+1}(s) = \max_q\{(\varepsilon\pi(q) + V_n(s - \varepsilon q))(1 - \mu_{n+1}(s, q)) + V^I(s)\mu_{n+1}(s, q)\} \tag{44}$$

Which defines the strategy $\eta_{n+1}(s)$ and the payoffs $V_{n+1}(s)$ and $W_{n+1}(s)$.

By the induction hypothesis, we can run n from 1 to N, and the lemma holds for any induction step $n \leq N$. ■

The next step is to prove that each game ends in finite time. The intuition is clear. The resource is finite, and as the buyer requires at least the outside surplus level, so the resource must be exhausted in finite time.

Lemma 4 *The number of periods after which the game $\Omega(s_0, N)$ stops is bounded from above by some M^* that is proportional to $\frac{s_0}{\varepsilon}$ and M^* is independent of N for N sufficiently large.*

Proof. Let $M > 0$ be the number of periods before stopping. For the buyer to continue in the first period, its surplus must at least be equal to the surplus after immediate investment. As the post-investment surplus is increasing in the remaining stock, we have the inequality

$$\sum\nolimits_{i=1}^{M} \varepsilon u(q_{t_n}) - M\varepsilon\overline{u} \geq 0,$$

Let ϕ be an upper bound for $u(q)/q$ (if no such upper bound exists, take an arbitrary $\delta \in (0, \bar{u})$ and define ϕ for $[u(q) - \delta]/q$; the proof is adjusted in an obvious way). Notice that $\sum_{i=1}^{M} \varepsilon q_{t_i} < s_0$. Substituting gives

$$\phi s_0 > M\varepsilon\overline{u}.$$

We can thus take $M^* = \frac{\phi}{\overline{u}} \frac{s_0}{\varepsilon}$. Thus, any game such that $N > M^*$ stops before M^*. ■

When N becomes sufficiently large, and ε approaches zero, the game still ends in finite time: $T \leq M^* \varepsilon = \frac{\phi}{\overline{u}} s_0$. We now prove convergence of the strategy functions when N becomes sufficiently large. The intuition is clear. For each $\varepsilon > 0$, when the game has to end within M^* periods, then it will not matter whether there are N or more periods available as long as N is sufficiently larger than M^*. To refer to a game for which N is large enough so that a further increase does not matter, we introduce the term "long game". For the proof of the lemma below, we find that we need some slack in the lenght of the game. Therefore, we use the wording "long game" for games that satisfy $N \geq M + M^*$. The next lemma says that "long games" are identical.

Lemma 5 *Let the game $\Omega(s_0, N)$ be a "long game", i.e., it stops in M periods and satisfies $N \geq M + M^*$. Then, the SPE outcome is identical in all games $\Omega(s_0, N')$ where $N' \geq M + M^*$.*

Proof. By the lemmas above, we know that for given N there exists a unique SPE and some finite stopping period $M \leq M^*$. The proof is by induction on the equilibrium stopping period M.

Let $\Omega(s_0, N)$ stop at some M. For such pair (N, M), define S_M^N as the set of all initial stocks s such that the stopping period for $\Omega(s, N)$ is exactly M. Clearly, $s_0 \in S_M^N$. Define $Q_M = \cup_{N' \geq M + M^*} S_M^{N'}$ as the set of initial stocks for which there is a long game that stops exactly in M periods. Thus, Q_M collects all the stocks and long games that lead to stopping in exactly M periods. Clearly, $S_M^N \subseteq Q_M$, but not necessarily $S_M^N \supseteq Q_M$. If $s' \in Q_M$ but $s' \notin S_M^N$, then some long game $\Omega(s', N')$ stops after M periods, while $\Omega(s', N)$ does not stop after M periods, and the lemma cannot be true. Thus, we must have $Q_M = S_M^N$ for the lemma to hold.

Define $R_M = \cup_{M' \geq M} Q_{M'}$ as the set of stocks such that there is a long game that stops in at least M periods. Finally, let $\Sigma_M = Q_M \backslash R_{M+1}$. Thus, Σ_M is the set of stocks for which M is the maximum stopping time among all long games. For the lemma to hold, we must have $S_M^N = \Sigma_M$ and also the pay-off equivalence must hold.

The induction hypothesis is labeled $\mathcal{P}(M)$, and it contains the following items:

(i) If for some initial stock s_0 a long game stops exactly in M periods, then all long games stop after exactly M periods: for all N with $M + M^* \leq N$ we have $S_M^N = \Sigma_M$.

(ii) If for some initial stock s_0 a long game stops after exactly M periods, then pay-offs and strategies are the same for all long games: for all N, N', s_0 with $s_0 \in \Sigma_M$, $M + M^* \leq N, N'$ we have $V_N(s_0) = V_{N'}(s_0), W_N(s_0) = W_{N'}(s_0), \eta_N(s_0) = \eta_{N'}(s_0), \mu_N(s_0, q) = \mu_{N'}(s_0, q)$.

By induction, we show that if $\mathcal{P}(0), .., \mathcal{P}(M-1)$ hold, then $\mathcal{P}(M)$ holds as well for all $M \leq M^*$.

Consider $\mathcal{P}(0)$. If $s_0 \in \Sigma_0$, then clearly $M = 0$ for all $N \geq M^*$. The induction hypothesis is trivially satisfied.

Assume $\mathcal{P}(0), .., \mathcal{P}(M-1)$ holds. We show $\mathcal{P}(M)$ in three steps:

(a) $\Sigma_M \subseteq S_M^N$ for all $N \geq M + M^*$;

(b) $s \in \Sigma_M$ implies: $V_N(s) = V_{N'}(s)$, $W_N(s) = W_{N'}(s)$, $\eta_N(s) = \eta_{N'}(s)$, $\mu_N(s, q) = \mu_N(s, q)$ for all $N, N' \geq M + M^*$;

(c) $S_M^N \subseteq \Sigma_M$ for all $N \geq M + M^*$.

Consider step (a). Let $s \in \Sigma_M$, $N \geq M^* + M$, and assume that game $\Omega(s, N)$ stops at some $k \leq M - 1$. By $\mathcal{P}(M-1)$, the game stops at k for all $N \geq M^* + M - 1$. This is a contradiction with $s \in \Sigma_M$. We must thus have $s \in S_k^N$, where $k \geq M$. But if $k > M$, $s \notin \Sigma_M$. Hence, $s \in S_M^N$.

Consider step (b). Let $s \in \Sigma_M$, and consider $N, N' \geq M^* + M$. By step (a), we have $s \in S_M^N$ and $s \in S_M^{N'}$. Take the SPE strategy $\eta_N(s)$, and note that $s - \varepsilon \eta_N(s) \in S_{M-1}^{N-1}$ by definition. Since we have $N \geq M^* + M$, then also $N - 1 \geq M^* + M - 1$, and we can invoke $\mathcal{P}(M-1)$ to conclude that $\Sigma_{M-1} \subseteq S_{M-1}^{N-1}$ and that the continuation payoffs are stationary. The same applies to N'. The buyer's payoff is

$$\varepsilon u(\eta_N(s)) + W_{N-1}(s - \varepsilon \eta_N(s)) \geq W^I(s).$$

Use

$$W_{N-1}(s - \varepsilon\eta_N(s)) = W_{N'-1}(s - \varepsilon\eta_N(s)),$$

to conclude that the offer $\eta_N(s)$ is accepted in the game $\Omega(s, N')$ and, conversely, the offer $\eta_{N'}(s)$ is accepted in the game $\Omega(s, N)$. Thus,

$$\eta_N(s) \in \Gamma_{N'}(s), \eta_{N'}(s) \in \Gamma_N(s).$$

Similarly, the seller's continuation payoff satisfies

$$\begin{aligned} V_{N-1}(s - \varepsilon\eta_N(s)) &= V_{N'-1}(s - \varepsilon\eta_N(s)) \\ V_{N-1}(s - \varepsilon\eta_{N'}(s)) &= V_{N-1}(s - \varepsilon\eta_{N'}(s)). \end{aligned}$$

None of the two offers can yield strictly higher payoff to the seller. Thus, the seller must make the same offer, and $V_N(s) = V_{N'}(s)$, $W_N(s) = W_{N'}(s)$, $\eta_N(s) = \eta_{N'}(s)$, $\mu_N(s, q) = \mu_{N'}(s, q)$.

Step (c) is proven by contradiction. Assume $S_M^N \neq \Sigma_M$. Consider the set for which there is some long game that lasts strictly longer than M periods: $[0, s_0] \setminus \cup_{k \leq M} \Sigma_k$. Now let s' be an element in this set such that the continuation stock $s'' = s' - \varepsilon q$ is not in the same set for any feasible supply $q > 0$. It follows that there exists a game $\Omega(N', s')$ that ends in $m > M$ periods with $N' \geq M^* + m$. By construction, $s'' = s' - \varepsilon\eta_{N'}(s') \in \cup_{k \leq M} \Sigma_k$. Because of $s'' \in S_{m-1}^{N'-1}$, and the properties of $\mathcal{P}(0), .., \mathcal{P}(M-1)$ we cannot have that $s'' \in Q_{M-1}$, and thus $s'' \in \Sigma_M$ and $m = M + 1$. But (b) ensures that the pay-off functions are stationary on Σ_M, and it follows that the buyer in the game $\Omega(N, s'')$ will accept the same offers as for the game $\Omega(N', s'')$, and similarly, that the seller will have indentical best responses. Therefore, $\eta_N(s'') = \eta_{N'}(s'')$, and we must have $m = M$, which contradicts $m > M$. ∎

8 Appendix: Alternative timing of stages

We consider the case that the buyer has to decide on the investment decision before observing the seller's supply, and we will show that under this timing the equilibrium degenerates. Let s^* be again the resource level at which investment takes place. We will show that no $s^* < s_0$ can exist, under stationary strategies. Notice that post-investment pay-offs $W^I(s^*)$ and $V^I(s^*)$ do not change. As the buyer does not observe supply before deciding, the decision to invest can only depend on the stock $d_t = \mu(s_t)$. Thus, s^* is the highest value less than s_0 for which $\mu(s_t) = 1$. Both the buyer and the seller know s^*. The seller's supply decision is not constrained by the buyer's outside option, as long as $s_t > s^*$. To maximize profits, the seller will solve a standard monopoly problem for an infinite horizon under the constraint $s_t > s^*$. This gives a supply policy $q_t = \eta^M(s_t - s^*)$. One feature of $\eta^M(s)$ is that it approaches zero when the stock approaches s^*, i.e., $q_t \downarrow 0$ when $s_t \downarrow s^*$. Now consider a point in time where the supply is so low that consumer surplus is below the long-run level: $u(q_t) < \overline{u}$. On observing s_t, the buyer knows the seller's supply. When investing immediately, the buyer's excursion pay-off is $W^I(s_t)$. When waiting, the pay-off is $\varepsilon[u(q_t) - \overline{u}] + e^{-r\varepsilon}W^I(s_t - \varepsilon q_t)$. The first term is negative, the second term is less then it's immediate payoff, thus waiting yields a lower pay-off. The buyer will invest. That is, for any s^*, the buyer will find it optimal to invest strictly before the stock has dropped to this level. The only possible outcome is immediate investment: $s^* = s_0$. The intuition is as follows. The change of timing removes all bargaining power from the buyer. The seller then must abuse leadership and provide too low supplies at some point in time. That is, the seller cannot bribe the buyer to wait. The buyer who knows this must invest immediately.

9 Appendix: Continued competition post-investment

Here we consider the case that the seller can continue to supply the resource after the arrival of the substitute. We will show that this option does not essentially alter the results. We will restrict the analysis to the case without discounting. Assume that the substitute is a backstop that can generate a flow q at fixed flow costs c plus constant marginal costs $\overline{p}$. Importantly, the costs c measures a part of costs that is independent of scale. The basic analysis in the main text can be interpreted as the case in which the substitute has only maintenance costs: $c > 0, \overline{p} = 0$. When the seller does not save any resource for the long-run state, consumer surplus satisfies $\overline{u} = u(\overline{q}) - c = \widetilde{u}(\overline{q}) - \overline{pq} - c$ with $\widetilde{u}'(\overline{q}) = \overline{p}$.

The seller can supply in the long-run state, but only at price $p_t = \widetilde{u}'(\max\{\overline{q}, q_t\}) \leq \overline{p}$. Flow profits are $\pi^L(q_t) = p_t q_t$, where we use superscript L for the long-run state. The profit is linear for low supply levels, $q_t \leq \overline{q}$, and strictly concave for $q_t > \overline{q}$. Without discounting, maximization of net present value profit $V^L(s_t)$ means that the seller maximizes prices, and thus, supplies do not exceed $\overline{q}$: $V^L(s_t) = \overline{p}s_t$. This strategy also implies that the buyer will not receive any surplus from the resource saved to the long-run state.

During the interim state, the long-run sales option suggests that the seller will not supply at prices below $\overline{p}$. Let $q^I \leq q^m$ be maximal supply during the interim state, defined by $\pi'(q^I) = \overline{p}$. The buyer's and seller's stopping pay-offs are now given by

$$W^I(s) = \begin{cases} k(u(s/k) - \bar{u}) \text{ if } s < kq^I \\ k(u(\overline{q}) - \bar{u}) \text{ otherwise,} \end{cases} \tag{45}$$

$$V^I(s) = \begin{cases} k\pi(s/k) \text{ if } s < kq^I \\ k\pi(q^I) + \overline{p}(s - kq^I) \text{ otherwise.} \end{cases} \tag{46}$$

Clearly, the long-run competition just reduces the treshold at which the scarcity cost starts to drive the equilibrium dynamics; nothing changes in our description if $\overline{p} = 0$ and $q^I = q^m$.

10 Appendix: Proof of Proposition 5

We prove Proposition 5, through a series of lemmas. The first lemma shows that supply is continuous at investment point. The second lemma uses this finding to prove that the seller prefers continuation to stopping at the investment point, which ensures that it is the buyer who decides on stopping, and thus (34) holds up to the point where stopping takes place. The third lemma then uses continuity of supply and (34) to establish the values for the resource stock and supply level at the stopping time. It also shows that the slope for (s, q) defined by (34) is downwards for values of s close to s^*, but upwards for large values of s.

Lemma 6 *Under constant elasticity of demand, the supply making the buyer indifferent,* $U(s) = U^I(s)$*, is continuous at the stopping time.*

Proof. Let q_T refer to optimal monopoly supply at stopping time T, and q_T^- is the supply just before. Continuity $q_T^- = q_T$ follows if $u'(q_T^-) = u'(q_T)$. Let $I(s_t, q_t) = 0$ be the reduced form of indifference equation (31). At stopping time T, the indifference condition cannot be extended to smaller s_t, which means that $I_q(s^*, q_T^-) = 0$. The condition gives $u'(q_T^-) = U'(s^*)$. We notice that $U'(s_t) = U^{I\prime}(s_t) = W^{I\prime}(s_t)$, so that for this lemma to hold we need to prove $u'(q_T) = W^{I\prime}(s^*)$.

Let $\lambda = \pi'(q_T)$. When the resource stock increases by small amount Δs, then the pertubation of the supply path Δq_t satisfies $\pi''(q_t)\Delta q_t = e^{r(t-T)}\Delta\lambda$, for some $\Delta\lambda$ such that $\int_T^{T+k} \Delta q_t dt = \Delta s$, that is, $\int_T^{T+k} \frac{e^{r(t-T)}}{\pi''(q_t)} dt = \Delta s/\Delta\lambda$. Let us use $\mu_t = \frac{\pi'(q_t)}{u'(q_t)} = \frac{q\widetilde{u}''(q_t)}{\widetilde{u}'(q_t)} + 1.$

We obtain

$$\begin{aligned} W^{I\prime}(s^*) &= \frac{\Delta W^{I\prime}(s^*)}{\Delta s} = \frac{\int_T^{T+k} e^{-r(t-T)} u'(q_t)\Delta q_t dt}{\int_T^{T+k} \Delta q_t dt} = \frac{\int_T^{T+k} e^{-r(t-T)} \mu_t \pi'(q_t)\Delta q_t dt}{\int_T^{T+k} \Delta q_t dt} \\ &= \frac{\int_T^{T+k} \mu_t \Delta q_t dt}{\int_T^{T+k} \Delta q_t dt}\lambda = \frac{\int_T^{T+k} \mu_t \Delta q_t dt}{\int_T^{T+k} \mu_T \Delta q_t dt} u'(q_T). \end{aligned} \tag{47}$$

The difference $W^{I\prime}(s^*) - u'(q_T)$ is caused by the difference in the average value of μ_t over the post-investment time interval $[T, T+k]$, and its value at time T. It is clear that, for utility with constant relative risk aversion, $W^{I\prime}(s^*) = u'(q_T)$. If utility has decreasing relative risk aversion, relative risk aversion will increase with decreasing q_t, and μ_t will increase, so that $W^{I\prime}(s^*) \geqslant u'(q_T)$. Similarly, if utility has increasing relative risk aversion, $W^{I\prime}(s^*) \leqslant u'(q_T)$. ■

Lemma 7 *Under constant elasticity of demand, the seller prefers continuation to stopping at the investment point.*

Proof. We will show that the seller's value function has a kink at the time of investment, $V'(s^*) > V^{I\prime}(s^*)$ when $W^{I\prime}(s^*) = u'(q_T)$, so the seller would always prefer continuation rather than stopping in such a situation. Changes in k play a role in the argument, and so we write the seller's payoff as a function of both the stock level and the transition time length k. We write $V^I(s,k)$ and $V^I(s)$ interchangeably, and similarly $V_s^I(s_t,k)$ and $V^{I\prime}(s_t)$. Flow profits are concave by assumption, and supplies strictly positive at the end of the overall sales time interval, $q_{T+k} > 0$. It is clear that the seller's value of the resource increases with the transition time length k, $V_k^I(s_t,k) > 0$. The value function satisfies the following Bellman equation

$$V^I(s^*,k) = \varepsilon\pi(q_T) + e^{-\epsilon r} V(s^* - \varepsilon q_T, k - \varepsilon). \tag{48}$$

Taking the limit for $\varepsilon \to 0$ (leaving k out of notation), we get

$$\pi(q_T) - rV^I(s^*) - q_T V_s^I(s^*) - V_k^I(s^*) = 0. \tag{49}$$

Thus, $\pi(q_T) > rV^I(s^*) + q_T V^{I\prime}(s^*)$. This together with continuous supply implied by Lemma 6 and value matching, $V(s^*) = V^I(s^*)$, implies $V'(s^*) = \pi(q_T^-)/q_T^- - rV(s^*)/q_T^- = \pi(q_T)/q_T - rV^I(s^*)/q_T > V^{I\prime}(s^*)$. ■

Lemma 8 *Given σ, assume k and r satisfy (with $\omega = r/(1-\sigma)$)*

$$\sigma(1-e^{-\omega k})^{\sigma} > 1 - e^{-\omega\sigma k}. \tag{50}$$

Then,

$$\begin{aligned} s^* &= \left[\frac{e^{-rk}\bar{u}}{(1-\sigma)^2 A^{\frac{-\sigma}{1-\sigma}} - (1-\sigma)rA}\right]^{-1/\sigma} \\ q^* &= A^{\frac{1}{\sigma-1}} s^* \end{aligned} \tag{51}$$

For $s \geq s^*$ but sufficiently close to s^*, seller's supply $q_t = \eta(s_t)$ is defined by (34) and declining in s_t. For s sufficiently large, $q_t = \eta(s_t)$ is increasing in s_t.

Proof. Recall that the buyer's indifference condition (34) is

$$I(s,q) = q^{\sigma} - \frac{e^{-rk}\bar{u}}{1-\sigma} - rAs^{\sigma} - q\sigma A s^{\sigma-1} = 0.$$

If the buyer is indifferent between between $d=0$ and $d=1$ at $s=s^*$, then we have $W^{I\prime}(s^*) = u'(q)$ by Lemma 6. Using the form in (33) for $W^{I\prime}(s^*)$, the condition $W^{I\prime}(s^*) = u'(q)$ gives

$$q^* = A^{\frac{-1}{1-\sigma}} s^*.$$

Condition $I(s,q) = I(s, A^{\frac{-1}{1-\sigma}}s) = 0$ then defines s^*, as given in (51).

Consider now the locus defined by $I(s,q)=0$ in the (s,q)-space. We have

$$\frac{dq}{ds}\Big|_{I(s,q)=0} = -\frac{-r\sigma A s^{\sigma-1} - \sigma(\sigma-1)qAs^{\sigma-2}}{\sigma q^{\sigma-1} - \sigma A s^{\sigma-1}}.$$

Note that $\frac{dq}{ds}\big|_{I(s^*,q)=0} = \pm\infty$, and that there are two solutions defined by $I(s,q)=0$ for $s > s^*$. Denote these by $q_L(s) \leq q_H(s)$, where the equality holds only at s^*. We argue that the equilibrium policy is

$$\eta(s) = q_L(s) \tag{52}$$

for $s > s^*$ close to s^*. To show (52), we consider the seller's dynamic program, given the buyer's rationality condition for continuation: the seller's optimal path is constrained by the requirement $q_t \in [q_L(s_t), q_H(s_t)]$. We show that $q_t = q_L(s_t)$ is the seller's optimal path for $s > s^*$ but close to s^*. Recall from (30) that

$$V'(s) = [\pi(q) - rV(s)]/q. \tag{53}$$

Maximizing the value of the resource is equivalent to maximizing $V'(s)$, given $V(s^*) = V^I(s^*)$. In the zero discounting case, the seller always supplies the lowest permissable quantity. With positive discounting, the seller will still supply the lowest permissable quantity if $dV'/dq < 0$ for all q above the minimum permissable level. We will check whether this condition holds at (s^*, q^*). If so, the seller will choose $q_L(s)$ rather than $q_H(s)$ close to the stopping point. Substitute $\pi(q)/q = \sigma q^{\sigma-1}$, and then the condition $dV'/dq < 0$, multiplied by q^2 reads

$$J(s,q) \equiv \sigma(1-\sigma)q^\sigma - rV > 0.$$

If $J(s^*, q^*) > 0$, the seller will choose the minimum permissable supply level for s_t close to s^*. Substituting from (32) and (37), we obtain that $J(s^*, q^*) > 0$ holds iff $1 - \sigma > rA^{\frac{1}{1-\sigma}}$. Rewriting as $(1-\sigma)^{1-\sigma} > r^{1-\sigma}A$, and expanding gives condition (50). That is, condition (50) ensures that $\eta(s) = q_L(s)$ for s close to s^*.

The above argument made clear that, as long as $J(s, q_L(s)) \geq 0$, the equilibrium path will satisfy (52). Now, if $J(s, q_L(s)) < 0$, the seller can maximize the (marginal) value of the resource by supplying $q_t = \left(\frac{rV(s_t)}{\sigma(1-\sigma)}\right)^{\frac{1}{\sigma}} > q_L(s)$. We thus find the equilibrium supply level for all $s > s^*$ to be

$$\eta(s_t) = \max\{q_L(s_t), \left(\frac{rV(s_t)}{\sigma(1-\sigma)}\right)^{\frac{1}{\sigma}}\} \tag{54}$$

Finally, we prove that $\eta(s_t)$ is increasing for sufficiently large s_t. Regarding the first term, we note that there exists an $\widetilde{s} > s^*$ with $I(\widetilde{s}, \omega\widetilde{s}) = 0$, and $q'_L(s) > 0$ iff $s > \widetilde{s}$. Function $q_L(s)$ is thus strictly decreasing on $[s^*, \widetilde{s})$ and increasing on $(\widetilde{s}, \infty)$.Regarding the second term, we note that $\frac{d}{ds_t}\left(\frac{rV(s_t)}{\sigma(1-\sigma)}\right)^{\frac{1}{\sigma}} > 0$. The intuition is that the seller's profit maximization defines a supply level that increases with the stock level for reasons similar to the Hotelling rule. Thus, for sufficiently large stock levels, irrespective of whether the seller prefers to sell more than needed to prevent the buyer from investing, or the buyer's indifference condition determines supplies, supplies will decrease when the stock is depleted. ■

The proposition in text is now proved by Lemmas 6-8.

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