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A SIMPLE FORMULA FOR THE SOCIAL COST OF CARBON

Abstract

The social cost of carbon (SCC) is the monetized damage from emitting one unit of CO_2 to the atmosphere, often obtained from computational Integrated Assessment Models (IAMs). We develop a closed-form formula that approximates the SCC for a general economy, and then explore the capacity of the analytical approach to capture the key SCC drivers and thus to replicate the results of the deterministic IAMs. The formula explains the parameter-driven SCC variation of a mainstream IAM without a systematic bias. The sensitivity analysis identifies and measures the performance limits of the closed-form formulas. We then use the analytic formula to structurally interpret a distribution of SCCs from deterministic IAMs, and develop an analytical breakdown and quantification of how different sets of parameters contribute to the SCC distribution. This allows the user of the formula to evaluate where particular parameter choices tend to place the resulting SCC outcome in the distribution of outcomes for the universe of deterministic IAMs.

Keywords: Climate change, social cost of carbon; integrated assessment models
Classification: Climate Change, Modeling

1. Introduction

The Social Cost of Carbon (SCC) monetizes the damage from releasing a ton of CO_2 to the atmosphere today. The monetization of damages is essential for the determination of optimal climate policies; pricing carbon according to the SCC provides the correct economic incentive for reducing current emissions. The SCC can be obtained by using computational Integrated Assessment Models (IAMs) that connect the global carbon cycle and temperature dynamics to a global economy description to assess the marginal welfare costs of emissions. There are several widely used IAMs.[1] While the IAMs overarch the contributions from various disciplines in climate-change research, they are not easily accessible to policymakers and researchers in general.[2] There are various systematic assessments of the assumptions in the IAMs and their effects on outcomes (Weyant, de la Chesnaye & Blanford, 2006; Hope, 2008; Nordhaus, 2008; Anthoff & Tol, 2013). The assessments show that higher climate sensitivity, higher estimates of damages for given temperature change, and lower discount rates generally lead to higher estimates for the SCC. They do not, however, solve a fundamental problem: to the wider audience, the IAMs remain a black box and the resulting SCC is a number accepted or rejected on the basis of trust or distrust in the models and their developers (Kelly & Kolstad, 1999a). Newbold et al. (2013) build a parsimonious and transparent IAM to help the user in understanding "how the SCC is likely to respond to alternative assumptions and input parameter values." Still, the user needs to ask the authors for the model, study it, run it, and analyze the outcomes.[3]

Golosov et al. (2014) derive an analytical formula for the SCC in an integrated assessment model, based on specific assumptions such as logarithmic utility and climate-change damages proportional to output and exponential in the atmospheric CO_2.[4] Gerlagh and Liski (2012) add a more comprehensive description of the climate system and associated temperature-change delays, and study the implications of the formula for the optimal policies in a general-equilibrium context with time-inconsistent preferences. In the current paper, we build on this emerging analytical literature to develop a closed-form SCC formula that approximates a general economy, and to provide a systematic testing of the

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[1] Most notable IAMs include DICE (Nordhaus W. D., 1992; 2008), CETA (Peck & Teisberg, 1992), PAGE (Hope, Anderson, & Wenman, 1993), MERGE (Manne & Richels, 2005), FUND (Tol, 2005), Webster et al. (2003), R&DICE (Nordhaus & Boyer, 2000).

[2] The proof of the pudding is in the eating. Here we consider accessibility as revealed through use by others. Most policymakers (need to) rely on supporting researchers who can run IAMs for policy assessments. Some IAMs are considered relatively simple, but only DICE (Nordhaus W. D., 1992) is sufficiently simple and comprehensive enough to have attracted a large group of users in the research community. R&DICE and FUND have publicly available descriptions and full source codes. R&DICE is used by a few researchers, but, to our knowledge, Ackerman & Munitz (2012) are the only researchers who used FUND, other than the developers. Learning to work with a model developed by someone else typically requires a very long learning time. Ackerman & Munitz (2012) reported on the results of their difficult process of running someone else's model; they required help by the developers.

[3] The current literature considers the existing simple models, such as DICE, as the furthest point to which one can get towards practical and accessible tools for assessment, away from large-scale 'black box' models, without sacrificing what is seen to be the essential structure for the climate-economy interactions.

[4] See Barrage (2014) for a sensitivity analysis of the assumptions.

formula. The objective is to explore the capacity of the analytical approach to capture the key SCC drivers and thus to replicate the results of the deterministic IAMs.

To evaluate the "internal validity" of the formula we test its performance against a mainstream numerical IAM (DICE, Nordhaus 2008).[5] Using a conservative sampling of the IAM parameters, we find that, on average, the formula explains the parameter-driven variation in the IAM SCC: the eight central parameters that enter the formula predict the IAM outcome, which depends on 14 parameters, without quantitatively significant systematic bias. The largest gaps in outcomes are associated with situations where climate damages are either strongly concave or convex, and, at the same time, the discount rate takes extreme values (low or high). The reasoning behind the deviations helps in understanding and measuring the performance limits of the closed-form formula.

To consider the "external validity" of the formula, we generate a distribution for the SCC from the underlying parameter distributions derived from the literature. The resulting distribution compares well with the existing distribution of SCC estimates produced by a sample of numerical IAMs (Tol, 2009). Since the formula is a structural interpretation for the SCC distribution, we can develop an analytical breakdown and quantification of how different sets of parameters contribute to the SCC distribution. The right-skewness of the SCC distribution has little to do with the carbon cycle and temperature delay parameters; damages and the determinants of discounting have a large contribution. In addition, due to the non-depreciating climate boxes, some climate impacts are permanent, fattening the tail of the SCC distribution when discounting falls towards zero. Importantly, analytical models without a multi-box description of the climate system ignore this tail-fattening effect.

In contrast with Golosov et al. (2014) and Gerlagh and Liski (2012), we derive the SCC in closed-form for a general economy whose development is approximated by a balanced-growth path. The approximation allows extending the formula to cover elements that have been noted important in the literature: non-unitary elasticity of marginal utility (Jensen and Traeger, 2014); climate-change damages increasing more or less than proportionally with income (Hoel and Sterner, 2007; Traeger, 2014); a climate-response function based on a more comprehensive emissions-temperature model (Gerlagh and Liski, 2012). The formal derivation thus requires a balanced-growth path; then, we test how the formula performs outside the balanced growth path.[6]

The current study should be understood as an investigation into the basic mechanisms of the numerical IAMs; we do not consider climate policy making under uncertainty or learning (e.g., Kelly and Kolstad, 1999b; Keller et al., 2004; Leach 2007, Crost and Traeger, 2013). Thus, the formula, as currently expressed, cannot provide guidance on how the optimal polices should develop over time

[5] Because of its public availability, conciseness, transparent documentation, and middle-of-the-road assumptions, we choose DICE (Nordhaus, 2008) for testing the accuracy of the formula. We extend DICE with damages that grow more or less than proportional with output, see footnote 22.

[6] In spirit, the approach is similar as in Nordhaus (1991); he considers a steady-state approximation.

when new information about the climate-economy interactions arrive (e.g., Lemoine and Traeger, 2014; Gerlagh and Liski, 2014), or how attitudes towards uncertainty might shape the current SCC (Jensen and Traeger, 2014).[7] [8] Instead, the objective is to link the predictions of the commonly used deterministic simulation models and those of the analytical representations for the current carbon price. With this focus in mind, the formula seeks to bring the knowledge that has been accumulated in the climate research, to the domain of analytical economics and further democratize it: by use of our formula, any reader can perform his or her own informed assessment about the SCC.[9] Given its performance, the formula can be seen as a useful policy tool. Without the need for assistance in running an IAM, it allows the policymaker to assess the sensitivity of the SCC estimate to climate sensitivity, climate-change damages, and discounting. That is, the formula directly shows an estimate for the SCC, given the choices for the set of fundamental parameters. Moreover, since we have evaluated how different parameter sets contribute to the SCC distribution, the user of the formula has tools for discussing where particular parameter choices tend to place the resulting SCC outcome in the distribution of outcomes for the universe of deterministic IAMs. For example, using median values for the carbon cycle parameters does not tend to place the estimate above or below the mean for the SCC outcomes; however, the median for damages places the output clearly below the mean SCC.

The paper is structured as follows. In Section 2, we introduce the climate-economy decision problem, and derive, without specifying the structure of the economy, a general expression for the SCC. We build on optimization, but the SCC expression turns out to be valid irrespective of whether the economy follows the optimal policy or not. The result allows us to obtain the closed-form SCC that approximates the general economy. We then run two types of experiments with the formula. In Section 3, we perform the sensitivity analysis of the formula against an extended version of DICE. In Section 4, we generate the SCC distribution and elaborate the sources of variation in the distribution. Section 5 concludes.

2. Model

2.1. Base model

We derive the SCC expression first for a general climate-economy model.[10] There is a representative consumer who maximizes the stream of future aggregate

[7] Rezai and van der Ploeg (2015) also take the simple formula from van den Bijgaart et al. (2013) to elaborate its validity under various extensions. Their paper is complementary to ours because, in particular, they use the formula to assess the time paths of the SCC in comparison with those produced by a benchmark model. They find minimal welfare losses if one applies the simple rule as the basis for the climate policy over time.

[8] The impact of short-term fluctuations on the choice of the optimal policy instrument has been considered, for example, in Hoel and Karp (2001, 2002) and in Karp and Zhang (2006).

[9] The reader can fill in the parameters and see the results immediately through an Excel file available through https://www.dropbox.com/s/mjtodx670mnf8bv/SCC%20tool%20v2.xlsx?dl=0.

[10] Golosov et al. (2014) provide formal conditions under which a simple formula is valid in a general equilibrium framework; the formula here deviates from their result as we present a

utilities, discounted at rate ρ. Population is denoted by L. Output F depends on capital K, emissions E, and the global average surface temperature T, and on time t that may capture technological development. Output is used for consumption C, replacement of depreciated capital $\delta_K K$, or net investments. Emissions add to the atmospheric CO_2 stock S, which depreciates at rate δ_S. Here, we define S as the CO_2 stock over and above the pre-industrial level of CO_2. Temperatures adjust at rate ε to their physical long-run equilibrium level $\varphi(S)$.

(1) $$max \int_0^\infty e^{-\rho t} LU(C/L)\, dt.$$

(2) $$C + \dot{K} = F(K, E, T; t) - \delta_K K$$

(3) $$\dot{S} = E - \delta_S S,$$

(4) $$\dot{T} = \varepsilon(\varphi(S) - T)$$

A dot denotes a time derivative. We suppress time scripts for variables, but keep the time script for production $F(.;t)$, to remind us that we assess climate change in a context of continued economic growth through technological change.[11]

The model assumes perfect foresight, and there is no uncertainty within the model; we assess the sensitivity of the SCC with respect to the parameters and do not assess the effect of within-the-model uncertainty on the policies. Demography, innovation and income growth may respond to environmental conditions, we neglect such feedback mechanisms and assume an exogenous innovation and population growth path. Emissions are endogenously determined; however, it is not obvious if changes in emissions are important for the level of the social cost in comparison to the contribution of the key parameter choices. We quantify the effect of policy choices on the SCC in our analysis.

We thus assume a continuous physical climate-change process. We seek to include a meaningful impulse-response functions that connect CO_2 emissions to atmospheric concentrations, and concentrations to temperature rise. For exposition, we postpone the full impulse-response to Section 2.3. We abstract from thresholds or tipping points where the dynamics of the carbon cycle or temperature adjustment change dramatically; see Lemoine and Traeger (2014) for further analysis.

Consider now the shadow-cost variables p, τ, χ for state equations (2)-(4), respectively. We interpret all shadow costs such that they take a positive value. That is, τ measures the marginal-utility weighted social cost of carbon – dividing by marginal utility, gives the monetized SCC that, when the optimal policy is implemented, equals the marginal product of energy use, $\partial F/\partial E = SCC$. Variable χ measures the current-value marginal cost of an increase in temperatures. In

richer description of damages depending on temperature change and income, time lags in climate change, and a non-unitary elasticity of marginal utility.

[11] Most IAMs assume implicitly or explicitly that both costs and benefits of emissions reductions are small compared to the economic benefits of technological progress over the relevant time scale (Azar & Schneider, 2002; Gerlagh & Papyrakis, 2003). That is, the decrease in $F(.)$ when emissions E drop to zero, or when temperatures increase by 3 degrees Celsius, is typically very small compared to the increase in F brought by innovation as captured through time t.

Appendix 6.1, we provide the Hamiltonian for the problem (1)-(4), and describe the first-order conditions in (40)-(44). These conditions include for C, E, K, S, T:[12]

(5) $$\frac{\partial U}{\partial C} = p$$

(6) $$p\frac{\partial F}{\partial E} = \tau$$

(7) $$\dot{p} = p\left(\rho - \frac{\partial F}{\partial K} + \delta_K\right),$$

(8) $$\dot{\tau} = (\rho + \delta_S)\tau - \frac{\partial \varphi}{\partial S}\varepsilon\chi,$$

(9) $$\dot{\chi} = (\rho + \varepsilon)\chi + p\frac{\partial F}{\partial T},$$

We note that (5) and (7) determine the optimal capital-investment versus consumption decision, while (8) and (9) are accounting equations that define the net present value of future marginal damages. The optimal climate policy is implemented through (6), defining $\partial F/\partial E$=SCC= τ/p. But note that we can calculate the SCC also for non-optimal climate policies. For example, if we substitute $\partial F/\partial E$=0 for (6), and maintain (5), (7)-(9), we find τ/p as the SCC for the business-as-usual scenario.[13]

For notational convenience, we write η for the (negative) elasticity of marginal utility, g for per capita consumption growth rate, r for the net rate of return on capital, and $R(s;t)$ for the consumption discount factor between time t and s:

(10) $$\eta \equiv -\frac{C\frac{\partial^2 U}{\partial C^2}}{L\frac{\partial U}{\partial C}},$$

(11) $$g \equiv \frac{\dot{C}}{C},$$

(12) $$r \equiv \frac{\partial F}{\partial K} - \delta_K,$$

(13) $$\frac{\dot{R}}{R} \equiv -r,$$

(14) $$R(s;t) \equiv R(s)/R(t).$$

We normalize $R(0)=1$, so that we can write for shorthand $R(s) \equiv R(s;0)$. Substituting the time derivative of (5) into (7) gives then the Ramsey rule:

(15) $$r = \rho + \eta g.$$

Using the notation above, we can rewrite (8) and (9) to derive an explicit formula for the SCC at time zero (see Appendix 6.1 for details) as the net present value of marginal damages:

(16) $$SCC(0) = -\int_0^\infty e^{-\delta_S t}R(t)\frac{\partial \varphi}{\partial S}(t)\int_t^\infty \varepsilon e^{\varepsilon(t-s)}R(s;t)\frac{\partial F}{\partial T}(s)\,dsdt.$$

[12] Note that $\partial F/\partial T$<0, so that the last term in (9) is negative, similar to the last term in (8).

This expression for the SCC continues to hold even when the policy choices are not optimal.[13]

2.2. Adding structure

We follow most of the IAM literature and assume that the relation between atmospheric CO_2 concentrations and equilibrium temperatures can be described through a logarithmic curve:

$$\varphi(S;c,m) = c\frac{\ln(1+S/m)}{\ln(2)}\ , \tag{17}$$

where c is the climate sensitivity parameter, that is, the temperature rise at a doubling of atmospheric CO_2, and m is the pre-industrial atmospheric CO_2.

The net output is gross output minus climate damages. Climate damages are assumed to increase with output changes, with elasticity ξ, and to increase with temperatures, with elasticity ψ:

$$F(K,E,T;t) = Y(K,E;t)\left[1 - \omega T^{\psi}\left(\frac{Y(K,E;t)}{L\bar{y}}\right)^{\xi-1}\right] \tag{18}$$

where $Y(.)$ is gross output before subtracting climate damages, and $\bar{y}$ is the reference per capita income level at which a one-degree temperature rise leads to relative damages ω.

The above functional form assumes that the costs of climate change are a smooth function of income, population, and temperature rise.[14] Most IAMs assume that damages are proportional to income. If the value of ecosystems lost by climate change increases more than proportional to income, $\xi>1$, the cost of climate change increases and we expect a higher SCC (Hoel and Sterner, 2007; Sterner and Persson, 2008). On the other hand, if economic growth allows society to cope more easily with the consequences of climate change, $\xi<1$, and we expect a lower SCC. Another typical assumption in IAMs is that damages are quadratic in temperatures, but some researchers suggest a higher or lower order damage function (e.g. Kopp & Mignone, 2013). Below we use quadratic costs as the median value for ψ; in the experiments we consider $1\le\psi\le4$.

Considering a climate system close to a stationary state, that is $T=\varphi(S)$, it is not immediately evident whether output damages are concave or convex in S – damages are given by a convex function of temperatures which in turn depend on S through a concave function. Indeed, for costs that rise quadratically with temperature change, $\psi=2$, the composite dependence of damages on

[13] Equation (16) is an accounting equation therefore it must hold for all optimal and non-optimal paths. Yet, obtaining a well-defined non-optimal path is not straightforward. Rezai et al. (2012) note that, in the representative agent framework, it is inconsistent to ignore the carbon price and, at the same time, to anticipate and internalize the impacts of capital investments, through induced emissions and climate change, on future production possibilities.

[14] Theoretically, (18) allows for net output to become negative. The purpose of this formulation is that it gives a simple analytical result. We compare the analytical results with those from a numerical model where damages are formulated such that output never becomes negative.

concentrations is close to linear over the domain where S is between 400 and 550 ppm[15]:

(19) $$\frac{\partial\left(c\frac{\ln(1+S/m)}{\ln(2)}\right)^2}{\partial S} \approx 1.3c^2/m.$$

Considering that the expected concentrations for the coming decades are in the range between 400 and 550 ppm, we use the average slope of the curve for our formula, and postulate the same approximation for other values of ψ. Writing $D = T^{\psi}$, we approximate the damage response as

(20) $$\frac{\partial D}{\partial S} \approx 1.3c^{\psi}/m,$$

(21) $$\frac{\partial F}{\partial D} = -\omega\left(\frac{Y(t)}{L\bar{y}}\right)^{\xi-1} Y(t),$$

(22) $$\frac{\partial F}{\partial T}\frac{\partial \varphi}{\partial S} = \frac{\partial F}{\partial D}\frac{\partial D}{\partial S} \approx -1.3\frac{\omega c^{\psi}}{m}\left(\frac{Y(t)}{L\bar{y}}\right)^{\xi-1} Y(t)$$

We foresee the following shortcoming of approximations (20)-(22). Consider an increasing temperature path. The formula assumes that marginal damages are constant over the range 400-550 ppm, but when ψ is high (>2), marginal damages are increasing with temperatures. Thus, the formula understates marginal damages in the long run where the temperatures are high; it overstates marginal damages in the short run where the temperatures are low. When the discount rate ρ is small, the long-run understatement of the damages becomes important and the formula SCC will tend to return a too low value. For high discount rate, the formula's overstatement of the shorter-term damages receives more weight, and then the formula tends to overshoot the true SCC. When ψ is low (<2), damages are concave and the approximation leads to opposite effects: shorter-term damages are understated and the longer-term damages are overstated. That is, we conjecture the formula to work best for values of ψ around 2, and a potential structural bias in the SCC formula when both discounting and the elasticity of damages with respect to the temperature are far from average.

To approximate the development of the economy, we consider a balanced growth path with constant savings rate, where the economy grows at constant growth rate g+l, with g the per-capita income growth and l the population growth rate. The climate-economy models do not typically satisfy the balanced-growth assumptions that effectively require all technological change to be labor-augmenting (Uzawa, 1961). Since the true economy does not follow a balanced growth path, the formula is meant to be an approximation to be tested.[16]

[15] At the time of writing CO_2 concentrations are about 400 ppm, which compared with a pre-industrial stock of approximately 275 ppm gives S/m=(400-275)/400≈0.45.

[16] Note that the closed-form formulas can also be obtained without the balanced growth assumption under specific structures for the preferences and technologies (Golosov et al. 2014).

Technically, however, we can use formula (16) to obtain the SCC for any growth path since the formula basically is an accounting equation.

Balanced growth ensures that the discount factor R decreases at constant rate $\rho+\eta g$. In the definition of the SCC (16), within the integrals we also find marginal damages, $\frac{\partial F}{\partial T}\frac{\partial \varphi}{\partial S}$, which equation (22) tells us grow at rate $\xi g+l$. We define σ the "climate discount rate", as the decrease in the value we attribute to future damages, corrected for the tendency of future damages to increase. Technically, σ is the negative overall growth rate of the terms within the integrals of (16), excluding the atmospheric CO2 depreciation δ_S and temperature adjustment ε:

(23) $$\sigma = \rho + (\eta - \xi)g - l.$$

Proposition 1. *Consider the economy* (1)-(4), *approximated by a balanced growth path with a constant population growth, and a constant per capita income growth. Assume damages that have a constant elasticity with respect to temperatures ψ and with respect to output ξ. The approximate social costs of carbon, as defined by expression (16), is then given by the reduced form formula:*

(24) $$SCC = \frac{1.3\omega c^{\psi}}{m}\frac{1}{\delta_S+\sigma}\frac{\varepsilon}{\varepsilon+\sigma}Y,$$

where ω, c, ψ, m, δ_S, ε, are the primitives, Y is the current output, and σ depends on the primitives ρ, g, l, η , ξ, as in (23).

The proof follows from substituting the growth rates into (16) which gives

(25) $$SCC(0) = \frac{1.3\omega c^{\psi}}{m}Y(0)\int_0^{\infty} e^{-(\delta_S+\sigma)t}\int_t^{\infty} \varepsilon e^{-(\varepsilon+\sigma)(t-s)}\,ds dt$$

(26) $$= \frac{1.3\omega c^{\psi}}{m}Y(0)\frac{1}{\delta_S+\sigma}\frac{\varepsilon}{\varepsilon+\sigma}.$$

There are no restrictions on σ to ensure that it is strictly positive. If σ is sufficiently negative, the SCC is without bound.[17] In this situation however, the simple formula also loses relevance. If the SCC grows very large, future abatement options become important.[18] Note also that, as long as information regarding the appropriate parameter values is not updated, the SCC is expected to grow at the rate of income, *Y,* although we do not intend to consider SCC time

[17] For (23), this would be the case either if $\sigma < -\delta_S$ or $\sigma < -\varepsilon$. In Section 2.3 we consider a refinement with more detailed climate dynamics. As these dynamics account for the fact that a very small share of emissions remain in the atmosphere for more than a 1000 years, the SCC is without bound already for $\sigma = 0$.

[18] For a high SCC value, it becomes profitable to capture CO_2 from the air. The trade-off is then not so much between future benefits of preventing climate change and present costs of reducing emissions, but between the latter and the future costs of CO_2 air capture. The, the policy will be determined by the lowest-cost abatement strategy instead of the tradeoff between emission cost and benefits.

paths in this paper.

An intricate part of the formula (24) is in the last two terms. The first term, $1/(\delta_S+\sigma)$, measures the economic lifetime of atmospheric CO_2. Both a rapid carbon depreciation, through high δ_S, and a high discount rate, σ, reduce the economic lifetime of CO_2. When there is no discounting, σ=0, $1/\delta_S$ measures the mean lifetime of atmospheric CO_2, about 50 to 100 years. For a 2 per cent annual climate discount rate, the *economic* lifetime of atmospheric CO_2 drops to a level between 1/(.02+.02)=25 and 1/(.01+.02)=33 years.

The second term, $\varepsilon/(\varepsilon+\sigma)$, measures the carbon price discount related to the delay of damages caused by the earth's heat inertia. An immediate full temperature adjustment, $\varepsilon\to\infty$, results in no discount. Slower adjustment implies that the impact of increases in atmospheric CO_2 is more distant, which reduces the carbon price. For a typical 2 to 4 percent annual temperature adjustment speed, and an annual 2 per cent climate discount rate, the delay discount factor lies between .02/(.02+.02)=0.5 and .04/(.02+.04)=0.67. The delay discount factor can be approximated by a temperature lag. Suppose temperature change is lagged by N years after the corresponding change in the atmospheric CO_2 stock, and the discount rate is σ, then the lag results in a discount factor $e^{-\sigma N}$ for the net present value of damages. If we substitute N=25 years, and consider a discount rates of 2% per year, we find that $X=e^{-0.5}=0.61$, which is within the range 0.5-0.67 stated above. A simplified interpretation of the 2 to 4 percent temperature adjustment speed is thus that temperature change lags about 25 years behind atmospheric carbon dioxide concentrations.

Jointly, the terms $1/(\delta_S+\sigma)$ and $\varepsilon/(\varepsilon+\sigma)$ weigh the persistence and delay of climate change; they cumulate the damage response over time, with weights decreasing exponentially at rate σ. Through these terms the SCC formula approximate the connection between emissions and damages in the IAMs.

2.3. Extension of the climate dynamics

The simple model assumes a single depreciation factor for the atmospheric CO_2 and a single temperature adjustment speed. We use the simple model in testing the formula's performance against DICE in Section 3. In this subsection, we extend the simple model to allow for a more flexible representation of the carbon cycle and temperature adjustments.[19] The extension allows us to quantitatively assess the contribution of climate system parameters to the carbon price distribution in Section 4. Thus, the simple and extended models have different roles in the quantitative exercise; the former identifies the key parameters necessary for matching the DICE outcomes, and the latter provides an extension connecting to the wider literature. Moreover, the extension shows that the simple climate description used in the previous Section, by assumption, puts a bound on the contribution of discounting to the carbon price.

In the extension, the atmospheric CO_2 depreciation is described through a set of impulse response functions with exponential decays, where each function

[19] We have also considered other extensions while maintaining a closed form solution. For example, we can allow for a declining, instead of exponential, population growth. However, this extension turned out to be less important for the quantitative evaluation than the climate system description.

is labeled by $i \in I = \{1, \dots, n\}$, and a_i is the share of emissions with decay rate δ_i, as in Maier-Reimer and Hasselmann (1987) and Hooss et. al. (2001):

$$(27) \qquad S(t) = \sum_{i \in I} S_i(t),$$

$$(28) \qquad \dot{S}_i(t) = a_i E(t) - \delta_{Si} S_i(t),$$

In the Appendix, we present 16 carbon-cycle models as proxied by Joos et. al. (2013) through an ensemble of exponential decay functions (see Figure 7). All models show a rapid decay in the first decade, and most models suggest that a substantial fraction of CO_2 remains in the atmosphere after 1000 years.[20]

In analogy to the atmospheric carbon depreciation that is represented through a multi-response function, temperature change can be represented through a multi-temperature response function (Caldeira & Myhrvold, 2013). The more general concentration-temperature response function then becomes:

$$(29) \qquad T(t) = \sum_{j \in J} T_j(t),$$

$$(30) \qquad \dot{T}_j = \varepsilon_j (b_j \varphi(S; c, m) - T_j)$$

with $\sum_{j \in J} b_j(t) = 1$. We can now establish

Proposition 2. *For the same assumptions as in Proposition 1, but with a multi-response function for atmospheric CO_2 and temperature change, the social costs of carbon is given by the reduced-form formula:*

$$(31) \qquad SCC(0) = \frac{1.3\omega c^{\psi}}{m} Y(0) W(\sigma, a, \delta_S) X(\sigma, \varepsilon),$$

$$(32) \qquad W(\sigma, \boldsymbol{a}, \boldsymbol{\delta_S}) = \sum_{i \in I} \frac{a_i}{\sigma + \delta_{Si}}; X(\sigma, \boldsymbol{\varepsilon}) = \sum_{j \in J} \frac{b_j \varepsilon_j}{\sigma + \varepsilon_j}$$

The formula is derived in a similar manner as equations (22) and (23) for the one-box model. For interpretation, note that $W(.)$ measures the economic life-time of emissions. When the climate discount rate approaches zero, $\sigma \to 0$, $W(.)$ becomes

$$(33) \qquad W(0, \boldsymbol{a}, \boldsymbol{\delta_S}) = \frac{a_1}{\delta_{S1}} + \dots + \frac{a_n}{\delta_{Sn}}.$$

Thus, the economic life-time of CO_2 becomes the physical life-time of CO_2. In particular, if a share of emissions, say $a_i > 0$, depreciates slowly, $\delta_{Si} \to 0$, term $W(.)$ becomes unbounded. This is an important difference to the simple model where, with vanishing discounting, the economic life-time of CO_2 converges to $1/\delta_S$. See also Gerlagh and Liski (2012) for further discussion. In Section 4, we

[20] The earth system models suggest that the fraction remaining in the atmosphere increases with cumulative emissions. Such can increase the SCC, an effect that we, as most IAMs, do not account for.

quantify how the climate system uncertainty, in the form of very low possible decay rates in some parts of the climate system, together with low discounting, translates into a tail-fattening effect in the SCC distribution.

Similarly, $X(\sigma, \boldsymbol{\varepsilon})$ is the discount factor associated with the slow temperature adjustment. We postpone the further analysis of this factor to Section 4.

3. Experiment I: testing the formula

We now evaluate quantitatively how well the formula predicts the SCC of DICE (Nordhaus, 2008).[21] The experiment is conducted by assuming distributions for 14 key climate and economic parameters entering DICE, and then sampling 100,000 draws for the parameter vector. Each draw defines also the subset of parameters that enter our formula. In the analysis, our dependent variable is the difference between the formula SCC and the DICE SCC (or, the SCC gap); the independent variables are the parameter realizations. We evaluate the contributions of various parameters to the SCC gap.

3.1. Sampling procedure

The 14 parameters for DICE describe the climate sensitivity, damage severity, the structure of time preferences, population growth, income growth, baseline emissions, and abatement costs.[22] The full list of the parameters is provided in the Appendix (Table 4), together with the quantitative values that we obtain from the literature. Specifically, we use the values in the literature to present each parameter by a right-skewed distribution, taken to be log-normal and distinct for each parameter. This set of log-normals defines our primitive parameter distributions, used also in the analysis of Section 4. However, in this Section, for the purpose of setting a conservative test for the formula's performance, we want to oversample the extreme parameter realizations far from the median. To this end, for each parameter, we transform the primitive log-normal to a log-uniform distribution, while matching the median of the original distribution; effectively, the sampling is from a uniform distribution applied to the logarithm of the parameters. This sampling procedure oversamples the corners of the original parameter space, compared to the primitive distribution.[23]

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[21] DICE is the single-most used IAM. To the knowledge of the authors, DICE is the only IAM that satisfies three conditions: (i) the source code is publicly available and can be run easily, (ii) for each major version of the model, an integrated and complete model description is publicly available, (iii) it is convenient in use. For other IAMs, either the model code is unavailable, or the model descriptions are scattered over various publications, or the model is built using software for which few researchers have the required skills.

[22] The damage specification in DICE implicitly assumes ξ, the rate of increase of marginal damages with income, is equal to unity. Here, we extend the DICE damage specification to allow for $\xi \neq 1$.

[23] This transformation would be unnecessary if it was numerically possible to cover all corners of the parameter space. Below, the sensitivity analysis suggests that the current sampling proceduce has sufficient coverage of the parameter space: the estimated parameter contributions to the gap between DICE and the formula remain stable as we include larger subsets of parameters.

There are eight parameters that enter our formula: climate sensitivity (c), relative damages at 3K temperature increase (ω), damage-temperature elasticity (ψ), damage-output elasticity (ξ), elasticity of marginal utility (η), time discount rate (ρ), consumption growth rate (g), and population growth rate (l).[24] [25] To save on the dimensions of the parameter space, we do not consider variations in the parameters of the carbon cycle when testing the performance of the formula; for the contribution of the "natural science" parameters to the carbon price, see Section 4. For the experiment in this Section, we use a one-box approximation of the climate system, assuming that 25% of CO_2 emissions decay very rapidly, and 75% decays at 1 per cent per year. The temperature adjustment process in our experiment formula assumes that 25% of temperature adjustment is reached immediately, while the remaining 75% of the temperature adjustment happens at 1 per cent per year. The two factors W and X of Section 2.3 become:

$$W(\sigma) = \frac{0.75}{\sigma+0.01}; X(\sigma) = 0.25 + \frac{0.0075}{\sigma+0.01}. \quad (34)$$

In the following charts, each dot presents an outcome from one parameter draw.

3.2. The choice of the benchmark: climate policy effect

Before testing the formula we should ask if the formula outcome should be tested against the DICE SCC with optimal climate policies or against the DICE SCC in the business-as-usual scenario. The optimal climate policy has an effect on the DICE SCC level – the effect depends on the shape of the damage curve. One may conjecture that a higher order damage exponent, ψ >2, tends to lead to damages that are convex in concentrations, while a smaller exponent imply concave damages. For a convex damage curve, marginal damages are increasing in S. Hence, a cut in emissions results in a reduction in the SCC, so that an active climate policy (a first-best scenario) lowers the SCC as compared to a business-as-usual scenario. Similarly, for ψ<2, one may conjecture that the optimal policy increases the SCC.

To make an informed choice, we first quantify the above effect of the climate policy on the SCC in DICE. For each parameter draw, we calculate two scenarios. The first scenario assumes a baseline policy without emission reductions. The second scenario is based on optimal policy, implying that abatement options and their development over time also enter the SCC calculations.

Figure 1 shows the policy impact on the SCC, calculated as the relative change [SCC policy – SCC nopolicy]/SCC nopolicy. The Figure confirms that for low values of ψ (<1.5), climate policy increases the SCC, as damages are a concave function of emissions, and thus lower emissions lead to higher marginal

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[24] The first six have direct counterparts in DICE. For the last two, the growth rates are not constants in DICE; we use the average growth for the first 50 years to obtain the corresponding parameter in the formula.

[25] Illustrating a feature in DICE that is not included in the formula, we note that DICE describes an autonomous decarbonization of the economy and the availability of abatement technologies and their costs.

costs. For high values of ψ (>2), climate policy decreases the equilibrium marginal costs of emissions. For very convex damages (ψ=4), responsive climate policy reduces the marginal costs of emissions on average by 40%. For climate damages that are quadratic in the temperature rise, the equilibrium carbon price is relatively insensitive to policies, with an average decrease of <10% brought by optimal climate mitigation policies. This result is not a surprise: Nordhaus (2008) reports the SCC both under the baseline and optimal policy scenario and finds that optimal policy reduces the SCC by less than 5 percent.

[* FIGURE 1 HERE ***]**

Figure 1: Climate policy effect on the SCC. Efficient climate policies reduce (increase) the SCC for large (low) values of ψ. On the vertical axis, the relative gap in the SCC between the climate policy scenario and the baseline with no policies. Each dot presents one parameter draw. The figure shows also the moving median, p5 and p95 lines. Dark dots present observations overlaying each other. Black dots indicate areas with more than 10 observations per square of 0.012x0.005.

The average change, over the full sample, of the SCC brought by climate policy is 15%.[26] The change is relatively large for SCC values far from the median. With this observation in mind, we note that in spirit our formula is closer to gauging the no-policy SCC than the policy SCC: the formula has no policy variable. We thus use the business-as-usual DICE SCC as our benchmark in the analysis.[27]

3.3. Testing the formula

We look first at the values of the outcome variables, that is, the SCC values predicted by the formula and DICE. In Figure 2 (left), we show the full set of outcomes on log-scale. The SCC outcomes are clustered along the 45-degree line, with a correlation of 0.985: there is a close association between the relative changes of the outcomes. Figure 2 shows the center of the distribution (median) as the solid line, and the 5 and 95 per cent cutoffs of the cumulative distribution as the dashed lines.

[26] Let ge=policy/no_policy be the effect; 0.15 is the average value for $|ge-1|$.

[27] There are also reasons of analysis that rationalize the choice. In particular, to ensure that we cover the entire parameter space, parameter values far from the median are oversampled compared to a more realistic parameter distribution. Hence, the expected difference between policy and no-policy SCCs is smaller than implied by the sampling procedure. As we will see below, for the full range of parameter values, the SCC varies by factor 10.000, from 0.1 to 1000 €/tCO2, so that the effect of policy is small, compared to the effect of parameter variations. Also, on the relevant domain the interaction of the convex damages and concave temperature adjustment approximately cancel out.

[*** FIGURE 2 HERE ***]

Figure 2: The DICE and formula SCC. Each dot corresponds to one parameter vector realization with the horizontal and vertical co-ordinates presenting, respectively, the DICE and our formula SCC values for the year 2015, in 2010 Euros. Left panel: logarithmic scale. Right panel: absolute values, with highest values eliminated for exposition. Both graphs show also the moving median, p5 and p95 lines. Dark blue dots present observations overlaying each other. Black dots indicate areas with more than 10 observations per square of 0.02x0.02 (left) or more than 100 observations per square of 1x1 (right).

In Figure 2 (right), we show the same information for the raw values. For visibility, the Figure shows the observations having lower value than 500 €/tCO_2. The correlation between the absolute values of the outcomes is .920. The overall take-way from the two Figures is that the formula predicts the absolute level of the DICE SCC with a slight upward bias; the relative changes are closely connected. Moreover, the relative precision (log-scale) does not noticeably change when moving to extreme parameter draws while the absolute prediction error, naturally, depends on the SCC level (absolute scale).

We turn to address the precise sources of the prediction error (the SCC gap), using classical regression analysis. We regress the log difference $[\ln(Formula\ SCC) - \ln(DICE\ SCC)]$ on the independent variables (parameters) to assess the contribution of each parameter to the gap. As usual, the log specification facilitates a percentage change (right-hand side variable linear) or elasticity interpretation (right-hand side variable in logs) of the estimated parameters. Specifically, we identify the parameters that explain majority of the variation in the gap by a stepwise inclusion of parameter sets in Table 1 below.

The first column reports the eight most important parameters for explaining the gap. They are introduced as linear terms in the regression, so that the reported coefficient gives the main effect of each parameter on the gap.[28] In the analysis, we subtract the mean from the parameter; the coefficients for the linear terms can be interpreted as the marginal effects at the mean value. Thus, for example, one per cent increase in the climate sensitivity leads to 0.26 per cent increase in the gap. The parameters reported in column 1 all enter the formula. The reported eight parameters explain 32 per cent of the variation in the gap (R2=.32). This is perhaps not surprising since the same parameters explain the majority of the variation also in the DICE outcomes (see Table 8 in the Appendix for the precise analysis of the explanatory power of the reported eight parameters for the DICE outcomes).

The second column introduces two additional terms: interactions ψ x ρ and ψ x η. We see that the explained gap variation doubles to R2=.64, while the estimated main effects in the first eight rows remain stable. These two interaction terms have the largest within-sample explanatory power (column 6, to be explained shortly) of all possible 14 linear and 91 interaction terms. This finding is consistent with the conjecture stated after equations (20)-(22). The

[28] As we take the gap in logs, and we know that the dependent variables are about linear in ln(c) and ln(ω), it is natural to transform these two parameters into their logarithms.

interactions are absent in our formula but they are relatively important for the DICE SCC (see Table 8 in the Appendix). Intuitively, the loss from not having the interactions in the formula is best understood by considering a high value for ψ (>2) combined with a low discount rate ρ. Marginal damages are increasing with temperatures over time; but this is not taken into account by the formula since marginal damages are assumed independent of the temperature levels. Thus, by not including the temperature dependence, the formula understates the damages in the long run which receives a high weight when discount rate ρ is below the mean discount rate (i.e., independent variable "discounting" is negative).[29] For ρ is above the mean value, the formula's overstatement of the shorter-term damages receives more weight, and then the formula overshoots the DICE SCC.

When ψ is low (<2), damages are concave and the mistake from not including temperature dependence in the formula leads to opposite conclusions: shorter-term damages are understated and the longer-term damages are overstated; discounting dictates which bias is important in the overall determination of the SCC gap. In Appendix 6.5, we provide a more detailed analysis of the parameter draws where the formula either over- or undershoots by factor 2: there is clear evidence that interaction ψ x ρ contributes strongly to the gap in these worst cases. Finally, most of the variation in the climate discount rate σ in (23) comes through the pure discount rate ρ and the elasticity of marginal utility η; the interaction terms ψ x ρ and ψ x η can be similarly interpreted.

The third column adds all remaining linear terms (14-8=6 parameters). This has no practical impact on the results; R2 and the previously reported effects remain stable. The fourth column adds all the remaining interactions, leading to the full set of parameters used in explaining the gap variation. The R2 increases to .82. We note that all reported parameter contributions remain stable as we move from left to the right, column by column, excluding the contribution of discounting. The movement of the estimated coefficient for the discount rate is suggestive of relevant interactions between the discount rate and the remaining parameters; however, individually, none of these stand out statistically or quantitatively important in a sense that we discuss next.

To gauge the potential of any given parameter (or interaction) to cause a large SCC gap, we report the spread of the parameter in the sample; that is, the fifth column reports the difference between the max and min values of the parameter (or, interaction) in the support. The final column then reports the gap caused when the parameter (or, interaction) moves from its mean value to its maximal or minimal value, to identify the most important contributions to the gap. This number is calculated as half the parameter spread multiplied with the estimated coefficient. Clearly, interaction ψ x ρ stands out. For intuition, moving from the mean ψ x ρ value to the min or max value of the interaction implies an increase of .7 in the gap, which, since the regression expresses the gap in logs, implies a factor two increase in the absolute value of the gap. None of the other reported (or non-reported) effects or interactions come close in quantitative magnitudes. The second-largest contribution comes from interaction ψ x η, and

[29] The interaction term ψ x ρ is negative (ψ above average, ρ below average), so the positive coefficient of 9.3 is consistent with the interpretation.

the third-largest, but by factor 2 smaller, comes from the climate sensitivity parameter. We thus expect that those cases where the gap between the formula and DICE will exceed a factor 2 will most likely be found in the far corners of ψ x ρ, which is confirmed by Figure 8 in Appendix 6.5.

Table 1: Relative gap between formula and DICE SCC values: dependence on main parameters.

	OLS gap (1)	OLS gap (2)	OLS gap (3)	OLS gap (4)	within-sample spread (5)	corner-center effect (6)
ln(c)	0.255	0.255	0.255	0.225	1.563	0.199
ln(ω)	0.043	0.044	0.044	0.045	3.219	0.072
ψ	-0.037	-0.037	-0.037	-0.038	2.900	0.055
ξ	0.202	0.202	0.202	0.226	1.263	0.143
ρ	1.733	1.692	1.693	1.093	0.075	0.041
η	-0.100	-0.100	-0.100	-0.103	2.478	0.127
g	-3.793	-3.825	-3.849	-3.313	0.109	0.032
l	10.13	9.894	9.886	10.39	0.005	0.029
ψ x ρ		9.279	9.278	9.803	0.142	0.694
ψ x η		0.192	0.192	0.194	4.582	0.446
Other linear var's	NO	NO	YES	YES		
Other interactions	NO	NO	NO	YES		
Nr independent vars	8	10	16	105		
Nr obs	100.000	100.000	100.000	100.000		
R2	0.316	0.640	0.642	0.815		

Note: All regressions include a non-reported constant. All reported coefficients are significant at p=0.01; t-values for reported coefficients are 10 or above. First 4 columns regress the gap between the formula SCC (log) and the DICE SCC (log). Column 5 presents the full spread of the independent variable in the sample (max–min). The last column multiplies the absolute value of the coefficient with the spread, to assess the maximum change in the gap within the sample explained by the independent variable.

With this background on the sensitivity analysis, we conclude the testing of the formula by plotting the raw value of the SCC gap on the level of the DICE SCC in Figure 3. It depicts the ratio of the SCCs plotted against the level of the DICE SCC. Over our parameter space, the SCC ranges by a factor 10.000, from 0.1 to 1000 €/tCO2. Throughout this range, in 90% of all observations the formula returns a value between 65% and 174% of the value calculated by DICE, with the average ratio between the two 1.04, and standard deviation of 0.36.[30] The figure shows more details: the tendency of the formula to exceed the DICE value at the high end of the distribution, and to fall short of the DICE value at SCC values close to 1 €/tCO2. Given the above sensitivity analysis, we evaluate that the largest part of the deviations arise from the non-linear relationships between CO_2 concentrations, temperatures, and damages that are not captured by the formula.

[30] The average of the natural log of the ratio equals –0.01, with standard deviation 0.30 with an overall 5-95% interval [–0.43,0.55].

[* FIGURE 3 HERE ***]**

Figure 3: The ratio of the SCCs. Each dot corresponds to one parameter vector realization with the horizontal and vertical co-ordinates presenting, respectively, the formula-DICE SCC ratio and the DICE SCC values for the year 2015, in 2010 Euros. Both axes have log scale. The figure shows also the moving median, p5 and p95 lines. Dark blue dots present observations overlaying each other. Black dots indicate areas with more than 10 observations per square of 0.02x0.01.

4. Experiment II: carbon price distribution

Our second experiment builds on the extended version of the formula that allows for a more flexible description of the climate system (Proposition 2 in Section 2.3). We conduct a Monte Carlo experiment as in Section 3; that is, we take 100,000 draws for the parameters entering the formula using right-skewed log-normal distributions.[31] In addition, in contrast with the experiment in Section 3 where the climate system was fixed, here we also sample the climate system parameters. The overall objective of this second experiment to generate, from the underlying parameter distributions, a carbon price distribution that is comparable to the distribution of outcomes from the IAMs in the literature. Since our representation of the SCC distribution builds on a closed-form formula, it allows us to provide a breakdown of how different sets of parameters such as those related to the climate system or damages contribute to the SCC distribution.

We start by introducing the sampling of the climate system parameters. Recall from Section 2.3 that carbon cycle parameters enter the formula through term $W(\sigma, \boldsymbol{a}, \boldsymbol{\delta_S})$ that measures the economic life-time of emissions; the temperature adjustment enters through the term $X(\sigma, \boldsymbol{b}, \boldsymbol{\varepsilon})$. We use 16 different models for the carbon cycle from Joos et. al. (2013), and 20 different models for the temperature adjustments from Caldeira & Myhrvold (2013); see Appendix 6.3. In the experiment, we randomly select one of the 16 carbon cycle models and one of the 20 temperature adjustment models. This defines a draw for the climate system.

Figure 4 presents the economic life-time of CO_2, $(\sigma, \boldsymbol{a}, \boldsymbol{\delta_S})$, as a function of the discount rate, for the ensemble of carbon cycles in Joos et. al. (2013) that we use in the analysis. The Figure shows the mean and the full support of $W(.)$ for a given discount rate. We see that for a discount rate of 3% per year, the economic life-time is approximately 20 years. For a discount rate of 1%, the economic life-time increases to 50 years. The variation between carbon cycles is small compared to the variation caused by the moving discount rate. Thus, Figure 4 suggests a limited economic meaning for the variation between carbon cycles.

[31] Recall that in Section 3, for the purpose of testing the formula, we translated the log-normals to log-uniforms. Here, since the testing is not the focus, we use the original log-normal distributions for the draws.

[* FIGURE 4 HERE ***]**

Figure 4: Economic life-time $W(\sigma,a,\delta_S)$ of atmospheric CO_2 as a function of the discount rate. Based on 16 models provided in Appendix 6.3.

Similarly, Figure 5 presents the discount factor $X(.)$ associated with the slow temperature adjustment and the ensemble of temperature adjustment models from Caldeira & Myhrvold (2013). For a discount rate of 3% per year, the discount factor is between .45 and .58. For a discount rate of 1% per year, the discount factor is between .68 and .8. Hence, omitting the temperature delay, as in Golosov et al. (2014), easily biases the carbon price by factor 2. This point has been elaborated also in Gerlagh and Liski (2012). The figure also shows that the variation between temperature delay models is more important, in economic terms, though the effect of changing the discount rate is still substantially larger.

Combining the two factors W and X, we see that a drop in the discount rate from 3%/yr to 1%/yr increases the factor WX from about 10 to about 40: a 2%/yr decrease in the discount rate increases the net present value of future damages by about 4.

[* FIGURE 5 HERE ***]**

Figure 5. Discount factor $X(\sigma, \varepsilon)$ for the net present value of damages because of the delay in temperature adjustment. Based on 20 models provided in Appendix 6.3.

Figure 6 depicts the density distribution of the SCC, obtained by sampling all parameters, as explained.

[* Figure 6 HERE ***]**

Figure 6: Density distribution of the SCC. Values are reported for the year 2015, in 2010 Euros. Tol's distribution comes from the database that supports his paper (Tol, 2009). SCC values in Tol were divided by 3.67 to convert 1995\$/tC into 2010€/tCO2, and then increased by 3% for each year between publication and 2015 to correct for the trend. Further information on the parameters' distributions is provided in Appendix 6.3.

The resulting distribution is strongly right-skewed with a median SCC of 20 €/tCO_2, mean 44 €/tCO_2, and more than 10 percent probability for a SCC higher than 100 €/tCO_2. A distribution from IAM outputs of 232 distinct studies results in a similar distribution when the numbers are converted to comparable units (Tol, 2009).

It is not straightforward to develop a statistical test for the goodness of the match with the distribution from the literature, given the elusive nature of this "data". We take it as given that Figure 6 is suggestive of consistency with the wider literature, and now we identify the determinants of the properties of the SCC distribution.

We want to identify measures for the first and second moments of the SCC distribution. We define a skewness measure (SM), equal to the relative gap between the expected or mean value E[.] and the median value M[.]. For a log-normally distributed variable, it is given by

(35) $$SM[X] = \frac{E[X]}{M[X]} - 1 = e^{\frac{1}{2}\sigma^2} - 1,$$

where $\sigma = sd[lnX]$ is the standard deviation of the underlying normal variable. Since a greater spread in an underlying variable translates into a greater standard deviation, the value of SM[.] is the increase in the expected value, relative to the median. It follows from the definition of the skewness measure and its formula for a lognormal distribution that, if the factors composing the SCC are lognormal distributed, then the SCC is lognormal distributed, and the SM for the SCC can be decomposed into the SM of its parts:

(36) $$Z = Z_1 \cdot Z_2 \text{ and } Z_1, Z_2 \sim logNormal \Rightarrow$$

(37) $$1 + SM[Z] = (1 + SM[Z_1])(1 + SM[Z_2])$$

The numbers in Table 2 provide some initial insight in the contribution of the SCC's parts to the gap between mean and median SCC. The table shows that the carbon cycle and temperature adjustment speed uncertainty have a low skewness measure.

But climate sensitivity, damages, and discounting each individually introduce considerable spread and right-skewedness to the SCC distribution. Furthermore, the table shows that, indeed, the skewness measure for the SCC is approximately equal to the (multiplicative) cumulative of its parts:

(38) $$SM[SCC] \approx (1 + SM[W])(1 + SM[X])(1 + SM[c^2]) \times \\ \times (1 + SM[\omega])(1 + SM[WX]) - 1$$

where $SM[W]$, $SM[W]$, and $SM[WX]$ denote the skewness measures of *W(.)*, *X(.)*, and *W(.)X(.)*, associated with the carbon cycle parameters, temperature adjustment parameters, and climate discount rate, respectively.

Table 2: Sources of SCC variation and skewness measures. Each row presents results from the Monte Carlo experiment, where only the first column parameters are varied. For the carbon cycle and temperature adjustment we estimated a median cycle (see Tables 4-6).

Source of variation	Median €/tCO_2	Mean €/tCO_2	St. deviation €/tCO_2	Skewness Measure
None	16.4	16.4	0	0%
Carbon cycle	31.3	32.8	3.2	+5%
Temperature adjustment	19.5	18.6	1.7	-5%
Climate sensitivity	29.6	38.9	30.4	+31%
Damage	29.3	39.3	31.8	+34%
Discount rate	29.5	34.1	18.9	+16%
All	20.2	43.9	75.5	+117%

The joint interaction of all uncertainties leads to a distribution where, as shown in Figure 6, the mean is twice as large as the median. The result is consistent with previous studies on the sensitivity (Hope, 2008; Nordhaus W. D., 2008; Newbold et al., 2013), but importantly, the formula helps to identify how the uncertainties for these parameters add up in shaping the SCC distribution. A one per cent increase in the skewness measure for the damage parameter translates into approximately a one percent increase in the SCC SM. Similarly a one percent increase in the climate sensitivity SM translates into approximately a 2 percent increase in the skewness measure of the SCC.

The formula also allows us to assess the sensitivity of the distribution to the annual discount rate as part of the distribution analysis (Table 3). The mean and median SCC increase by half when the discount rate, σ, falls from 3 to 2 percent, increase by factor 2 when the discount rate falls from 2 to 1 percent, but they increase more than eight-fold when σ falls from 1 to .1 percent. Due to the non-depreciating climate boxes, some climate impacts are permanent, fattening the tail of the SCC distribution when discounting falls towards zero. For a discount rate converging to zero, the expected SCC is without bound. This effect cannot be captured by analytical formulas having only one climate box.

Table 3: Discount rate sensitivity of the SCC.

Discount rate	Median €/tCO$_2$	Mean €/tCO$_2$	St. deviation €/tCO$_2$
0.1 %	280	511	698
1 %	35.7	63.5	83.8
2 %	18.3	32.6	43.0
3 %	12.3	21.9	28.9

Note: Each row presents outcomes from the Monte Carlo experiment, where only the discount rate is fixed.

5. Conclusion

This study offers a relatively simple, closed-form, formula for determining the SCC. We have derived the formula under a specific set of assumptions regarding economic growth, population growth and savings to provide an approximation of richer climate-economy descriptions. The formula is tested by comparison with a mainstream IAM.

In this exercise, draws are taken from parameter distributions for the key variables, of which some also enter the formula. A comparison then reveals that, despite its low informational requirement, the formula explains the parameter-driven variation of the SCC in DICE, the IAM used for the comparison. An application of the formula shows that for a parameter distribution, the formula generates a SCC distribution that comes close to that obtained in a comprehensive survey of previous SCC estimates.

The approach has limitations: it does not present an analysis of policy making under uncertainty. However, the results are quite informative about the basic mechanisms of SCC determination in deterministic IAMs. First, they imply that the SCC, as presented by the benchmark IAMs, is relatively independent of current or future policy choices and abatement options. The analytics demonstrates that only a few mechanisms are needed to understand the core of the determination of the SCC, as described by the mainstream IAMs.

Second, the analytic structure allows assessment of how different parameter sets contribute to the SCC value. Based on primitive parameter distributions, we found a strongly right-skewed SCC distribution, with a median of 20 €/tCO_2, mean 48 €/tCO_2, and a 10% probability of the SCC exceeding 100 €/tCO_2. The median (or best-guess) value for the SCC can readily be calculated using rule-of thumb values for the main parameters; however, the mean SCC is the more relevant measure for policymaking. Spread regarding the appropriate discount rate, climate sensitivity and damages mostly contributed to skew in the SCC distribution. The formula can easily be exploited to understand the effects of subjective choices on the deterministic SCC outcomes. In particular, the climate-system description with some fraction of carbon slowly depreciating, explains the effect of the discount rate: a reduction in the effective discount rate from 2% to 1% approximately doubles the SCC outcome, while the SCC increases more than 8-fold if this discount rate is reduced from 1% to 0.1%.

Finally, the formula indicates, and as has been noted in the literature, that the trajectory of the SCC is expected to increase approximately with income levels, as the size of the economy determines what is at stake. Yet, the scope of the current formula for such analysis is restricted since it excludes within-model parameter uncertainty (see Gerlagh & Liski, 2014, for analytical steps in this direction).

From a science-policy perspective, the formula answers to a call for a better connection between research in the climate-economics domain and the users of that knowledge (Gerlagh & Sterner, 2013). Without ignoring the insights gained in recent years on fat tails for damages, climate tipping points, and policy under uncertainty, much of the basic understanding about the cost-benefit analysis of climate policy is still close to the insights of the early 1990s. The formula captures some of these insights, enabling the stakeholders to reflect on the methods used to derive the SCC. By doing so, it can facilitate the communication between stakeholders and the research community.

6. Appendix.

6.1. The optimal control problem (1)-(4)

The current-value Hamiltonian for (1)-(4) reads

$$\begin{aligned}\mathcal{H} = LU\left(\frac{C}{L}\right) + p[F(K,E,T;t) - \delta_K K - C] \\ -\tau[E - \delta_S S] - \chi[\varepsilon(\varphi(S) - T)]\end{aligned} \quad (39)$$

Since we defined τ and χ to measure the negative value of the stock of atmospheric CO2 and global temperature change, respectively, the first order conditions are

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \quad (40)$$

$$\frac{\partial \mathcal{H}}{\partial E} = 0 \quad (41)$$

(42) $$\dot{p} = \rho p - \frac{\partial \mathcal{H}}{\partial K},$$

(43) $$\dot{\tau} = \rho \tau + \frac{\partial \mathcal{H}}{\partial S},$$

(44) $$\dot{\chi} = \rho \chi + \frac{\partial \mathcal{H}}{\partial T}.$$

After substituting the functional forms, we derive the FOCs (5)-(9). The FOCs (8) and (9), we can rewrite as

(45) $$\tau(t) = \varepsilon \int_t^\infty e^{-(\rho+\delta_S)(s-t)} \chi(s) \frac{\partial \varphi}{\partial S}(s) ds.$$

(46) $$\chi(t) = -\int_t^\infty e^{-(\rho+\varepsilon)(s-t)} p(s) \frac{\partial F}{\partial T}(s)\, ds.$$

In order to express the social costs of carbon as the net present value of marginal damages, we use (7) and identities (12), (13), (14), to connect the price deflator $R(.)$ to the marginal utility measure p:

(47) $$\dot{R}/R = \dot{p}/p - \rho \;\Rightarrow\; R(s) = cnst\; e^{-\rho s} p(s) \Rightarrow$$

(48) $$R(s,t) = e^{-\rho(s-t)} \frac{p(s)}{p(t)}$$

We can now rewrite (45) and (46) as

(49) $$\tau(t) = \varepsilon p(t) \int_t^\infty e^{-\delta_S(s-t)} R(s;t) \frac{\chi(s)}{p(s)} \frac{\partial \varphi}{\partial S}(s) ds.$$

(50) $$\chi(t) = -p(t) \int_t^\infty e^{-\varepsilon(s-t)} R(s;t) \frac{\partial F}{\partial T}(s)\, ds.$$

Substitution of t=0 in the first equation, substituting t for s, and substituting $R(t,0)=R(t)$, and $SCC(t)=\tau(t)/p(t)$, gives (16).

6.2. Parameters for the Monte Carlo experiment comparing the extended DICE model with the formula

We included 12 major parameters from DICE (Nordhaus, 2008) in our Monte Carlo parameter sample, and add the damage temperature elasticity and damage income elasticity. These parameters are listed in the table below. For each parameter, we derived distributions from the literature as stated in the last column of the table below. The central values are more or less in line with the typical values used for DICE, apart from the elasticity of marginal utility. Compared to the parameters listed for the sensitivity assessment for DICE (Table VII-1), we included the pure rate of time preference, the elasticity of marginal utility, the decline rate of labor productivity growth and decarbonization, and short- to long-term backstop costs. We excluded the fossil fuel resources and a transfer coefficient in the climate module. For consistency between the parameters and initial values, we recalibrated the DICE model with respect to the initial capital stock, productivity, population size and growth in the first decade 2005-2015.

Table 4: DICE parameter distributions.

Parameter [Units]	*Median*	*Lower cutoff value**	*Upper cutoff value**	*Source*
Climate sensitivity [K]	3	1.3719	6.5601	(a)
Damages at 3K (relative to output)	0.027	0.0054	0.135	(b)
Damage temperature power coefficient	2	1	4	
Damage income elasticity	1.15	0.67	2	(i)
Pure rate of time preference [yr^{-1}]	0.02	0.005	0.08	(c)
Elasticity of marginal utility	1.2	0.5	3	
Asymptotic size of population [mn]	10,000	7,300	13,699	(d)
Productivity growth [dec^{-1}]	0.154	0.109	0.218	(e, f)
Decline rate of productivity growth [dec^{-1}]	0.001	0.0005	0.002	(g)
Decarbonization rate [dec^{-1}]	0.073	0.0479	0.1113	(e, g, h)
Decline rate of decarbonization [dec^{-1}]	0.003	0.0013	0.007	(e, g, h)
Backstop price [USD/tC]	1,170	768	1783	(g, h)
Ratio initial to final backstop price	2	1.3122	3.0482	(g, h)
Decline rate of backstop price [dec^{-1}]	0.05	0.0275	0.0909	(g, h)

Parameter distributions are log-normal, truncated at 2 standard deviations from the median; *for truncated distribution. Sources: (a) Dietz & Asheim (2012); (b) Tol (2009); Gerlagh & Liski (2012); (c) Weitzman (2001); (d) UN (2011); (e) World Bank, World Development Indicators. (2012), (f) OECD, OECD Productivity Statistics. (2012), (g) Nordhaus (2008), (h) Solomon et al (2007), (i) Hoel and Sterner (2007).

The literature on climate damages deals with damages for a given temperature increase. To match distributions as suggested by this literature, we jointly estimate the damage parameter and damage temperature power coefficient as follows. We rewrite (18), normalizing ω as a measure for damages at 3 Kelvin temperature perturbation:

(51) $$F(K,E,T;t) = Y(K,E;t)\left[1 - \omega(T/3)^{\psi}\left(\frac{Y(K,E;t)}{L\bar{y}}\right)^{\xi-1}\right]$$

Using this formulation, we draw ω and ψ independently, and (31) becomes

(52) $$SCC(0) = \omega\frac{1.3c^2}{9m}Y(0)W(\sigma,a,\delta_S)X(\sigma,\varepsilon),$$

6.3. Parameters for the Monte Carlo experiment using the SCC formula

For the second experiment, we vary the parameters c, ω and σ, and use 16 alternative carbon cycle representations and 20 temperature adjustment models (see Tables 5 and 6) to calculate the SCC according to equation (30). The parameter Y in the formula is fixed at 66.2 trillion Euros. The parameters c, ω and σ are drawn from a lognormal distribution as specified in Table 4. The lognormal distributions are based on the literature as noted in the last column of Table 4, and chosen to reflect the fact that the dispersion regarding the appropriate parameter values is highly asymmetric, with greater dispersion for high values. Still, some very high (or low) values are deemed unrealistic. The latter is captured by our use of cutoffs..

We generated a Monte Carlo parameter set and derived the SCC using Stata; the full source code is available online through https://www.dropbox.com/s/b3gdhwhmhmxirms/BGKL%20SCC%20v10.zip?dl=0.

Table 5: SCC parameter distributions.

Parameter [Units]	*Median*	*Mean**	*Standard deviation**	*Lower cutoff value**	*Upper cutoff value**	*Source*
Climate sensitivity [K]	3	3.218	1.222	1.3719	6.5601	(a)
Damage parameter	0.003	0.004	0.0032	0.0006	0.015	(b)
Climate discount rate [yr^{-1}]	0.018	0.0224	0.0154	0.005	0.072	

Parameter distributions are log-normal, truncated at 2 standard deviations from the median; *for truncated distribution . Sources: (a) Solomon et al (2007); (b) Tol(2009) and Gerlagh & Liski (2012).

Table 6: Carbon cycle parameters.

Model	a_0	a_1	a_2	a_3	δ_{S1}	δ_{S2}	δ_{31}
NCAR_CSM1.4	0	0.367	0.354	0.279	0.0006	0.0353	0.1881
HadGEM2-ES	0.434	0.197	0.189	0.180	0.0433	0.0433	0.2550
MPI-ESM	0	0.586	0.183	0.231	0.0056	0.1106	0.1112
Bern3D-LPJ	0	0.515	0.263	0.222	0.0005	0.0218	0.2583
Bern3D-LPJ	0.280	0.238	0.238	0.244	0.0036	0.0260	0.2029
Bern2.5D-LPJ	0.236	0.099	0.385	0.280	0.0043	0.0171	0.3865
CLIMBER2-LPJ	0.232	0.276	0.490	0.003	0.0037	0.1494	0.1494
DCESS	0.216	0.291	0.241	0.252	0.0026	0.0275	0.2943
GENIE	0.215	0.249	0.192	0.344	0.0037	0.0254	0.2323
LOVECLIM	0	0.361	0.450	0.189	0.0006	0.0461	0.4384
MESMO	0.285	0.294	0.238	0.183	0.0022	0.0400	0.4965
UVic2.9	0.319	0.175	0.192	0.315	0.0033	0.0377	0.2632
ACC2	0.178	0.165	0.380	0.277	0.0026	0.0271	0.2686
Bern-SAR	0.199	0.176	0.345	0.279	0.0030	0.0252	0.2433
MAGICC6	0.205	0.253	0.332	0.210	0.0017	0.0455	0.3339
TOTEM2	0	0.203	0.700	0.097	0.00001	0.0089	63.29114
Median	0.220	0.279	0.278	0.222	0.0035	0.0507	0.2892

Parameters taken from Joos et al (2013), η_0=0 for all models. The median cycle has been determined based on the 16 individual models.

[* Figure 7 HERE ***]**

Figure 7. Airborne fraction of CO2 emissions for 16 models, as in Joos et al. (2013)

Table 7: Temperature adjustment parameters.

Model	b_0	b_1	b_2	ε_0	ε_1	ε_2
BCC-CSM1.1	0.235	0.352	0.412	1.447	0.162	0.007
BCC-CSM1.1(m)	0.303	0.334	0.363	1.678	0.116	0.008
CanESM2	0.458	0.245	0.298	0.469	0.037	0.003
CSIRO-Mk3.6.0	0.197	0.212	0.591	1.248	0.113	0.005
FGOALS-g2	0.333	0.227	0.44	0.621	0.036	0.003
FGOALS-s2	0.079	0.453	0.468	5.155	0.212	0.003
GFDL-CM3	0.181	0.284	0.535	1.342	0.139	0.005
GFDL-ESM2G	0.13	0.432	0.438	3.390	0.315	0.003
GFDL-ESM2M	0.16	0.385	0.455	2.688	0.242	0.004
INM-CM4	0.197	0.481	0.322	3.106	0.188	0.002
IPSL-CM5A-LR	0.216	0.394	0.39		0.062	0.002
IPSL-CM5A-MR	0.185	0.379	0.436	2.262	0.097	0.003
IPSL-CM5B-LR	0.292	0.316	0.393	2.075	0.114	0.006
MIROC5	0.259	0.384	0.356	1.565	0.212	0.004
MIROC-ESM	0.204	0.364	0.432	1.449	0.107	0.003
MPI-ESM-LR	0.278	0.315	0.407	1.106	0.149	0.006
MPI-ESM-MR	0.23	0.38	0.39	2.463	0.172	0.006
MPI-ESM-P	0.302	0.317	0.38	1.733	0.141	0.006
MRI-CGCM3	0.305	0.356	0.339	1.473	0.095	0.006
NorESM1-M	0.223	0.297	0.480	1.942	0.145	0.005
Median	0.2218	0.3306	0.4476	0.9787	0.1980	0.0036

Parameters taken from Caldeira & Myhrvold (2013). The median cycle has been determined based on the 20 individual models.

6.4. Explaining the DICE outcome using regression analysis

The sensitivity analysis looks at the gap log(formula SCC/DICE SCC). We can also regress the log(formula SCC) and log (DICE SCC) separately on the same right hand side variables; the results in text can also be obtained by merging the results of these two separate regressions. However, it is of some interest to see how the central parameters explain the levels; we present the results for DICE in the table below. We present in the table below all parameters that have a max within-sample effect of at least 2, meaning that their variation can cause a factor 2 change in the DICE SCC.

Column 1 shows that the eight most important parameters explain more than 90 per cent of the variation; the parameters are the same as in the main text. The most important interaction terms (column 2) include those in the text but also one additional interaction.

Table 8: DICE SCC value dependence on main parameters.

	OLS DICE (1)	OLS DICE (2)	OLS DICE (3)	OLS DICE (4)	within-sample spread (5)	Center-corner effect (6)
$\ln(c)$	1.890	1.889	1.889	1.888	1.563	4.37
$\ln(\omega)$	0.947	0.947	0.947	0.946	3.219	4.58
Ψ	0.0389	0.0385	0.0385	0.0393	2.900	1.06
ξ	0.585	0.661	0.661	0.689	1.263	1.55
ρ	-38.96	-39.95	-39.95	-40.51	0.075	4.57
η	-0.837	-0.838	-0.838	-0.854	2.478	2.88
g	4.570	6.600	6.612	7.585	0.109	1.08
l	47.28	49.21	49.22	49.45	0.005	1.15
$\ln(c)$ x ψ		0.797	0.797	0.796	2.718	2.95
ψ x ρ		-9.135	-9.134	-9.643	0.142	1.98
ψ x η		-0.188	-0.188	-0.192	4.582	1.55
Other linear var's	NO	NO	YES	YES		
Other interactions	NO	NO	NO	YES		
Nr independent vars	7	9	16	105		
Nr obs	100.000	100.000	100.000	100.000		
R2	0.918	0.977	0.977	0.983		

Note: All regressions include a non-reported constant. All reported coefficients are significant at p=0.01. First 4 columns regress the DICE SCC (log). Column 5 presents the full spread of the independent variable in the sample (max–min). The last column multiplies the absolute value of the coefficient with half the spread, and then takes the exponent, to assess the change in the SCC when moving from the center of the parameter space to the furthest corner for that parameter.

6.5. Sensitivity analysis: the parameters associated with extreme deviations

The sensitivity analysis in the text shows that interactions ψ x η and ψ x ρ are important explanatory variables for the gap between the formula and DICE outcomes. We complement the regression analysis here by collecting all realizations where the formula deviates from the DICE value by more than factor two. In Figure 8 below all observations where the formula presents less than half the DICE value are characterized by high ψ and low ρ (left panel). Observations where the formula more than doubles the DICE SCC are characterized by either high ψ and high ρ, or low ψ and low ρ. There is no other pair of parameters with such clear patterns. The interaction effect of the next most-important interaction, ψ and η, is too small to see a comparable pattern as below.

[*FIGURE 8 HERE ***]**

Figure 8. Projections of all 1081 observations with SCC formula < 0.5 SCC DICE (left panel) and all 2280 observations with SCC formula > 2*SCCDICE, on 2-parameter plane: ψ and ρ.

7. References

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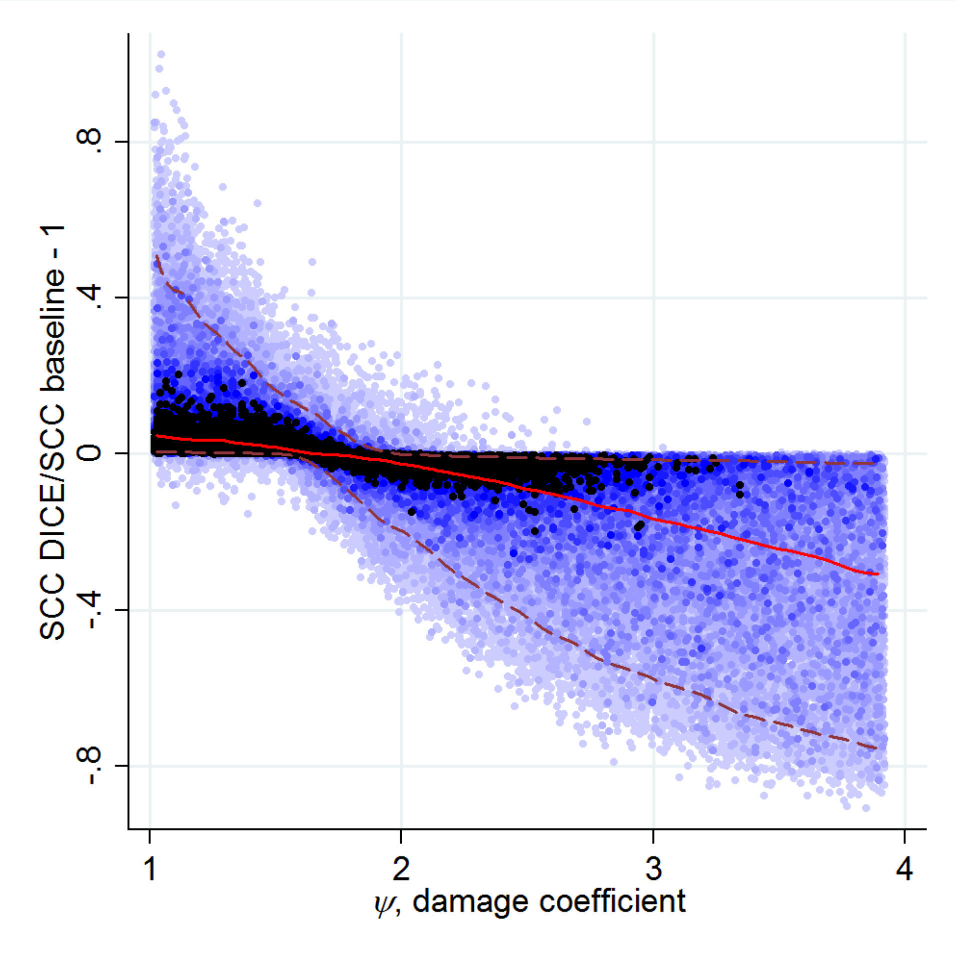

Figure 1: Climate policy effect on the SCC. Efficient climate policies reduce (increase) the SCC for large (low) values of ψ. On the vertical axis, the relative gap in the SCC between the climate policy scenario and the baseline with no policies. Each dot presents one parameter draw. The figure shows also the moving median, p5 and p95 lines. Dark blue dots present observations overlaying each other. Black dots indicate areas with more than 10 observations per square of 0.012x0.005.

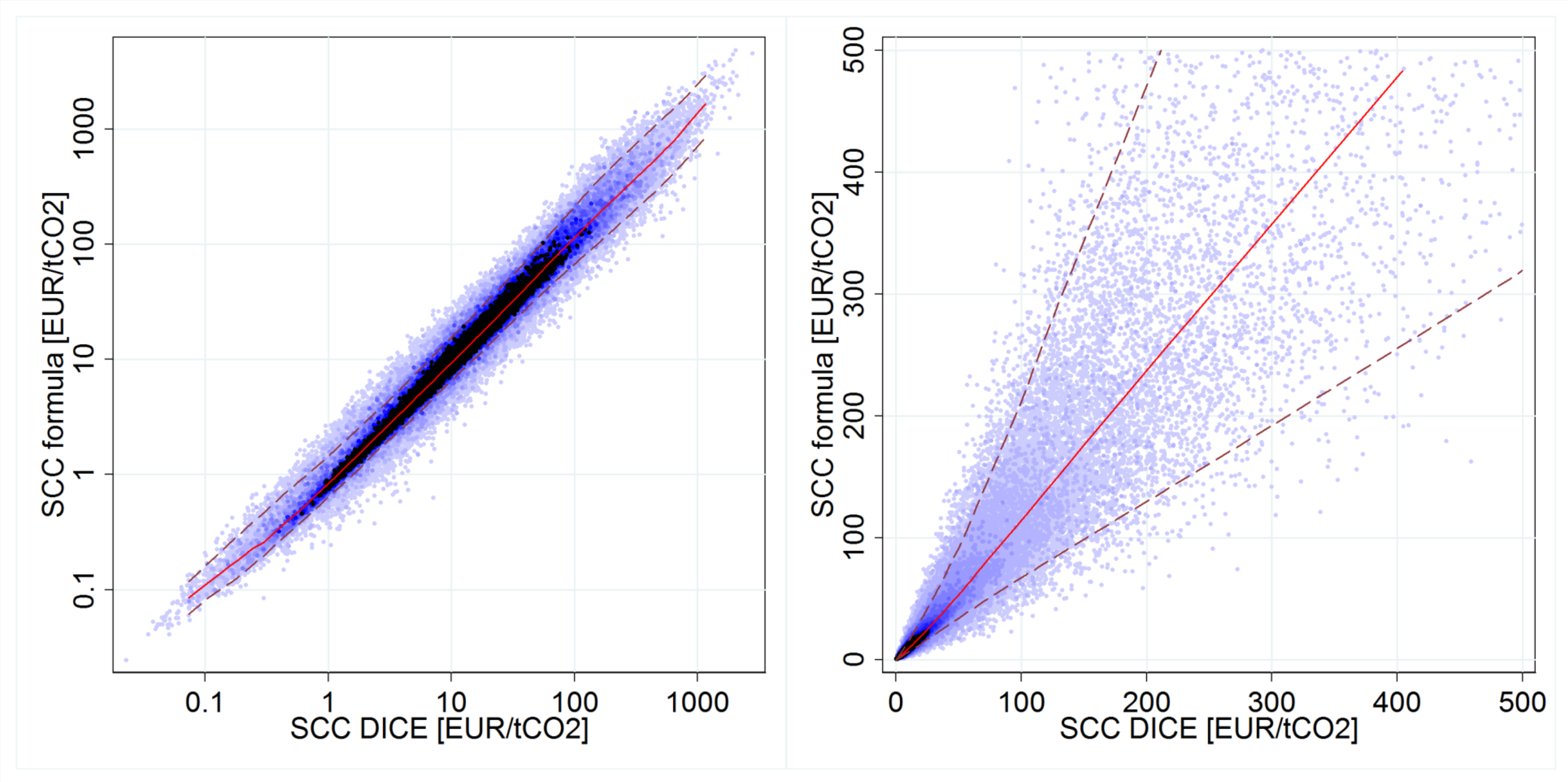

Figure 2: The DICE and formula SCC. Each dot corresponds to one parameter vector realization with the horizontal and vertical co-ordinates presenting, respectively, the DICE and our formula SCC values for the year 2015, in 2010 Euros. Left panel: logarithmic scale. Right panel: absolute values, with highest values eliminated for exposition. Both graphs show also the moving median, p5 and p95 lines. Dark blue dots present observations overlaying each other. Black dots indicate areas with more than 10 observations per square of 0.02x0.02 (log-scale left) or more than 100 observations per square of 1x1 (right).

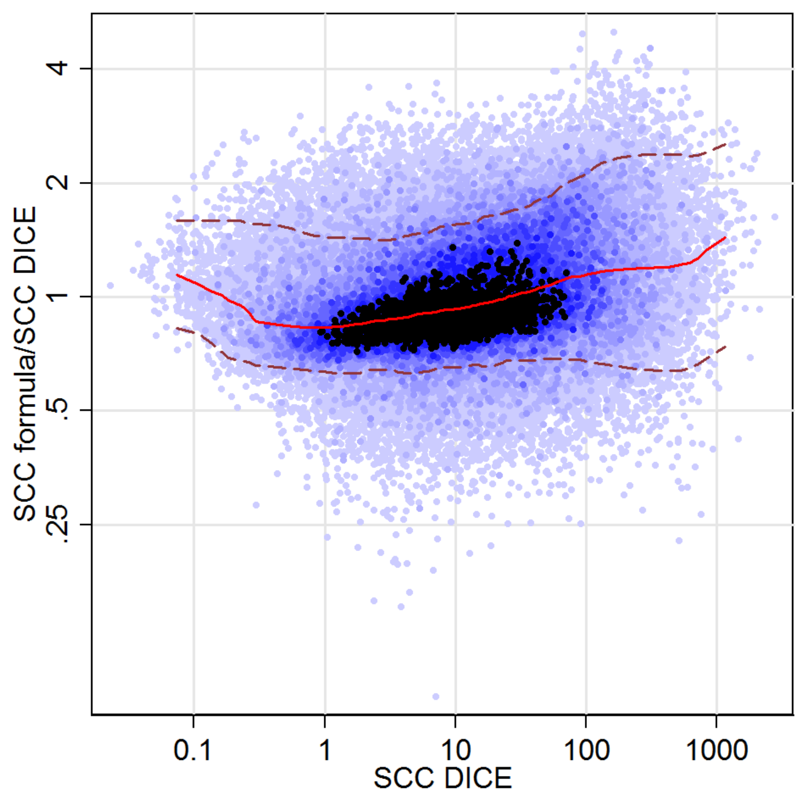

Figure 3: The ratio of the SCCs. Each dot corresponds to one parameter vector realization with the horizontal and vertical co-ordinates presenting, respectively, the formula-DICE SCC ratio and the DICE SCC values for the year 2015, in 2010 Euros. Both axes have log scale. The figure shows also the moving median, p5 and p95 lines. Dark blue dots present observations overlaying each other. Black dots indicate areas with more than 10 observations per square of 0.02x0.01 (log scales).

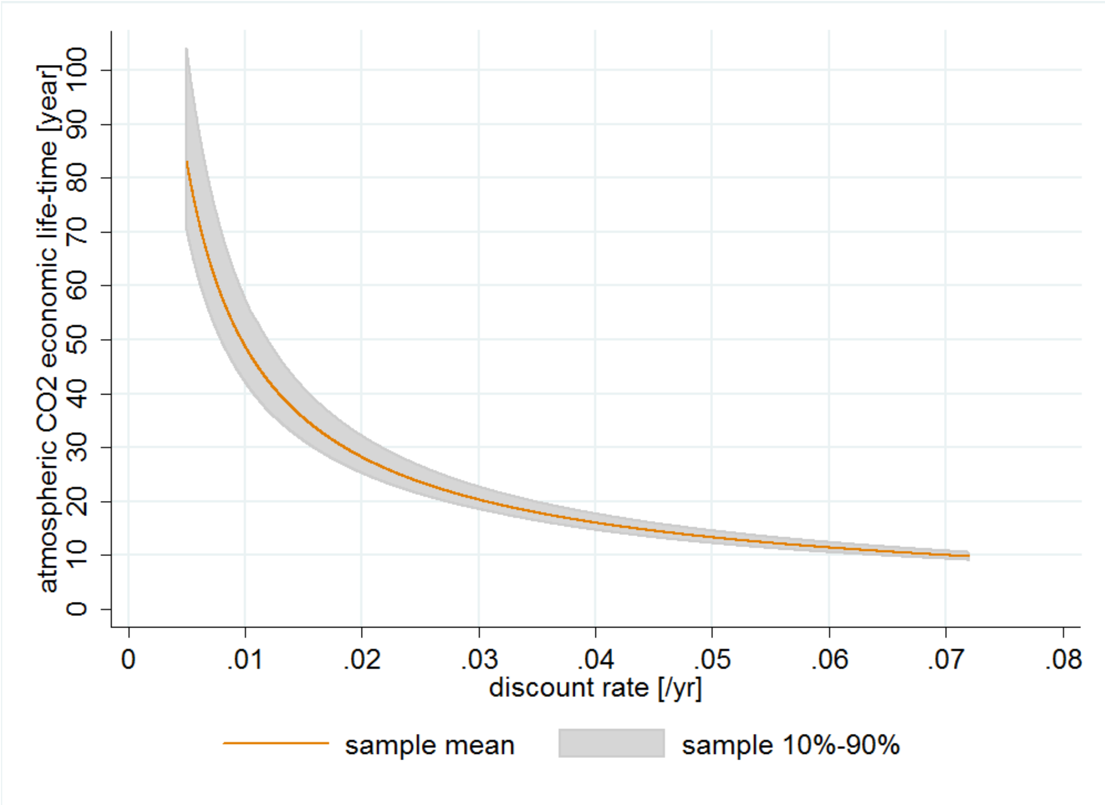

Figure 4: Economic life-time $W(\sigma, a, \delta_S)$ of atmospheric CO_2 as a function of the discount rate. Based on 16 models provided in Appendix 6.3.

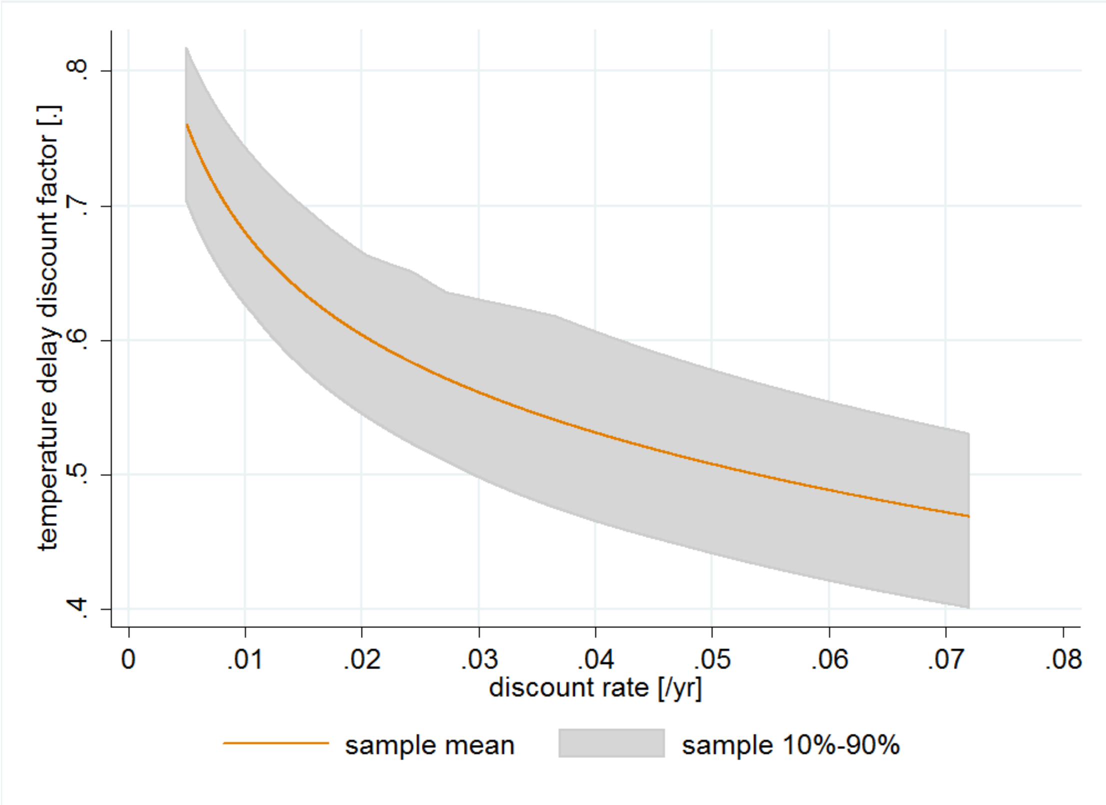

Figure 5: Discount factor $X(\sigma, \varepsilon)$ for the net present value of damages because of the delay in temperature adjustment. Based on 20 models provided in Appendix 6.3.

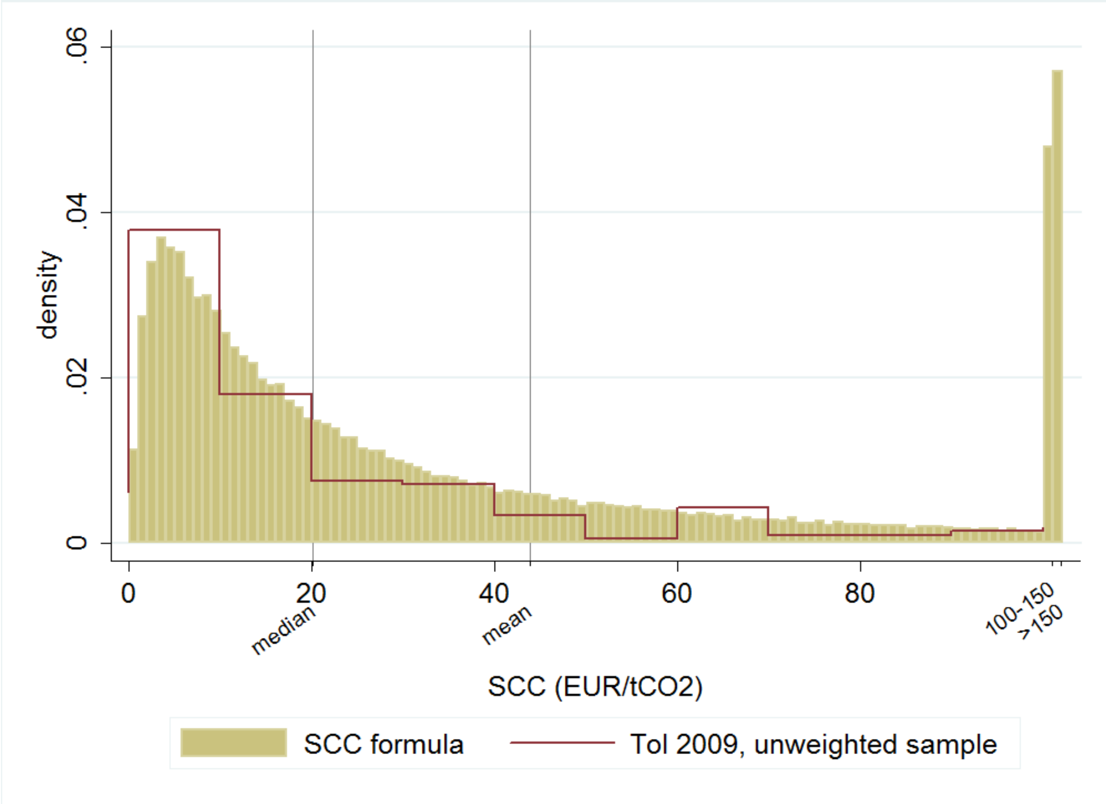

Figure 6: Density distribution of the SCC. Values are reported for the year 2015, in 2010 Euros. Tol's distribution comes from the database that supports his paper (Tol, 2009). SCC values in Tol were divided by 3.67 to convert 1995$/tC into 2010€/tCO2, and then increased by 3% for each year between publication and 2015 to correct for the trend. Further information on the parameters' distributions is provided in Appendix 6.3.

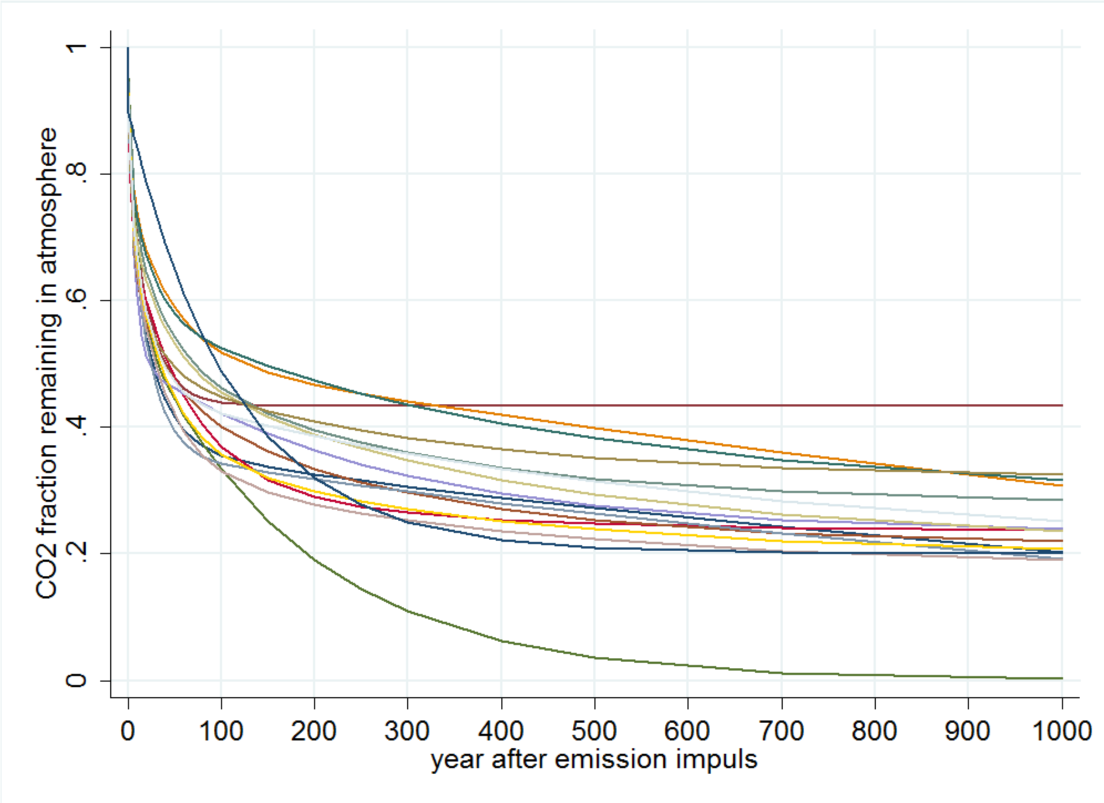

Figure 7. Airborne fraction of CO2 emissions for 16 models, as in Joos et al. (2013)

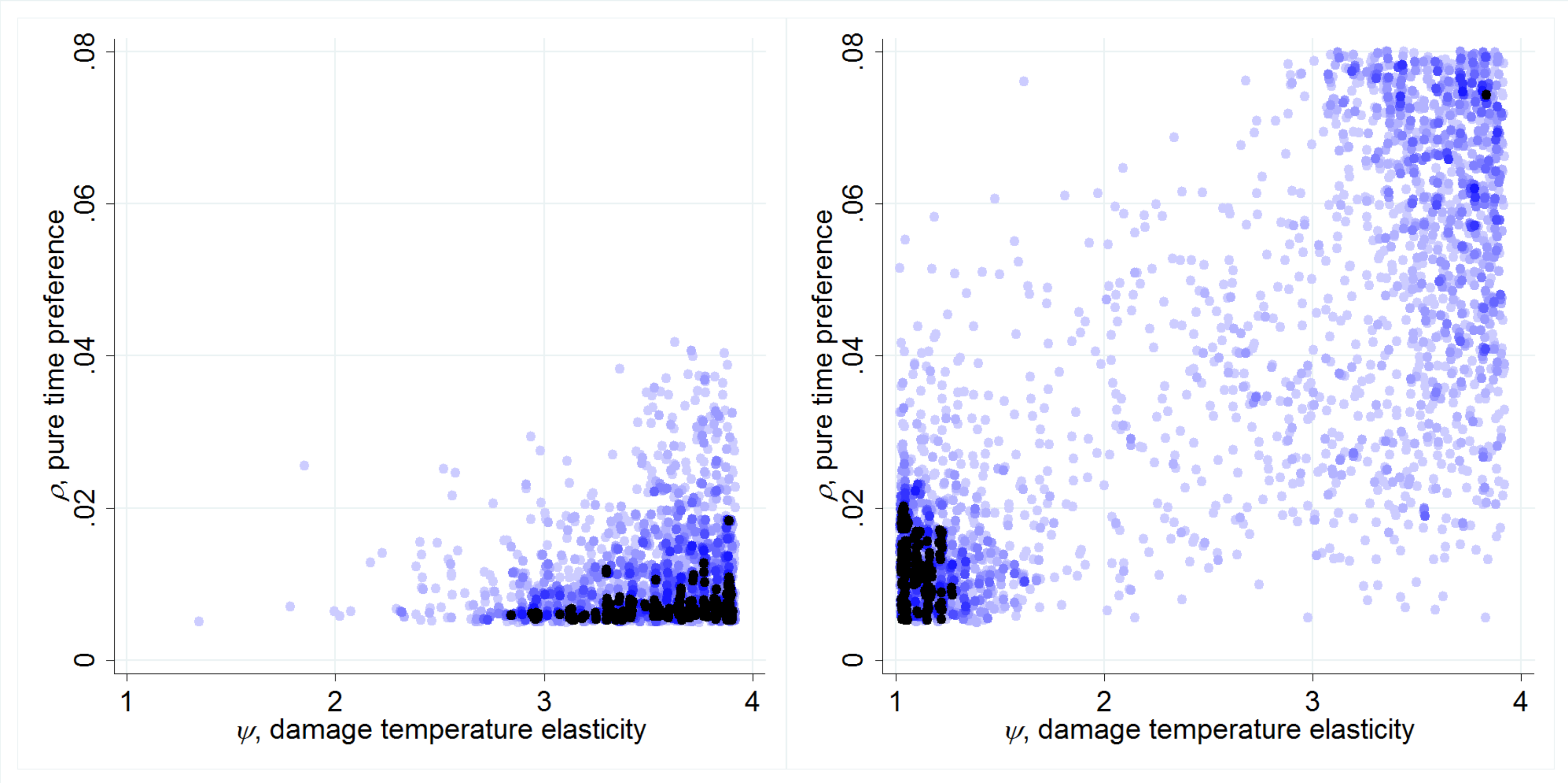

Figure 8: Projections of all 1256 observations with SCC formula < 0.5 SCC DICE (left panel) and all 2279 observations with SCC formula > 2*SCCDICE, on 2-parameter plane: ψ and ρ. Black dots indicate areas with more than 10 observations per square of 0.08x0.0016.